## MULTIPLE-ORDER DIFFRACTION GRATINGS WITH ASYMMETRIC PERIODIC PROFILE

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Abstract—A numerical analysis is presented for multiple-order diffraction gratings with asymmetric periodic profile. The advantages and disadvantages of this type of grating are demonstrated in comparison with its symmetric counterpart.

Multiple-order diffraction gratings stand out among diffraction optical elements for their ability to split a laser beam into several beams of equal intensity propagating at different angles (or having the intensities distributed in a chosen proportion). These devices have no analogs among conventional optical elements, and yet they are relatively simple to manufacture. The periodic structure of such a grating consists of rectilinear rulings of different width and can be synthesized on a standard photolithographic equipment. The profile of a period is rectangular and can be obtained by a single ion or chemical etching of the substrate.

Gratings have been proposed and fabricated [1, 2] as structures with rulings symmetrically placed about the centre of a period (Fig. 1a). In this case, K rulings per period imply K free parameters  $(\sigma_1 - \sigma_3)$  in Fig. 1a), that is, variables that may be varied independently to obtain the chosen intensity distribution in the orders of the diffraction pattern. If symmetry is abandoned and rulings on the right and left sides of the period are spaced independently, the number of free parameters will be 2K + 1 for the same K rulings (Fig. 1b). From this figure it is clear that the position of at least one ruling, say  $\sigma_1$ , should be fixed.

A greater number of free parameters naturally improves the control of light intensity in the diffraction orders. Theoretically, K rulings in a period ensure (K+1) equal orders on either side of the zero order for a symmetric period, and 2K orders for an asymmetric period. This means, including the zero order, N=2K+3 equal orders in the first case, and N=4K+1 in the second case. In practice, the maximum number of equal diffraction orders is not always attainable especially for an asymmetrical profile. Nonetheless, the symmetric profile increases the number of equal orders about 1.5-fold for the same number of rulings, or allows the number of rulings to be decreased in the same proportion if the number of equal orders is to be kept (Figs 2, 3).

For a grating with asymmetric profile, the intensity of light in diffraction orders

$$I_{b} = \frac{\sin^{2} \frac{\phi}{2}}{\pi^{2} l^{2}} (C_{1}^{2} + S_{1}^{2}), \quad l \neq 0;$$

$$I_{0} = 1 - 4Q(1 - Q) \sin^{2} \frac{\phi}{2};$$
(1)

where

 $I_1$  is the *l*th-order light intensity of the diffraction pattern,

$$C_{1} = \sum_{k=1}^{2K} (-1)^{k} \cos^{2} \pi l \sigma k;$$

$$S_{1} = \sum_{k=1}^{2K} (-1)^{k} \sin 2\pi l \sigma k;$$

$$Q = \sum_{k=1}^{2K} (-1)^{k+1} \sigma_{k};$$
(2)

 $\phi$  is the phase modulation over the grating profile:

$$\phi = \frac{2\pi}{\lambda} \Delta d(n-1),\tag{3}$$

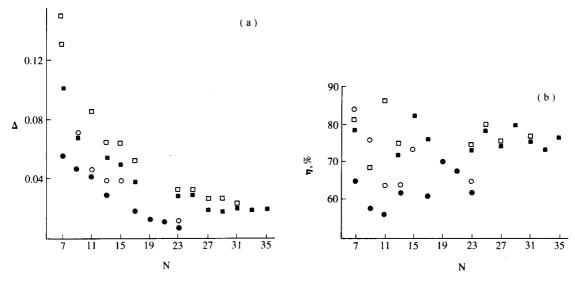


Fig. 4. (a) Width of minimum ruling and (b) overall efficiency in working orders of symmetric (circles) and asymmetric (squares) gratings with different numbers of equal-intensity orders. Open symbols represent solutions with the widest minimum ruling, and solid symbols stand for solutions with phase modulation  $\phi = \pi$ .

where

 $\Delta d$  is the profile depth of the profile,

n is the refractive index of grating material,

 $\lambda$  is the wavelength of light.

If the profile is symmetric, then for all k

$$\sigma_k = 1 - \sigma_{2k+1-k} \tag{4}$$

and Eqs (1) and (2) reduce to the familiar expressions [2].

This method of grating design is virtually the same as that of the symmetric case [2], but its interactive program is more efficient and fast in obtaining solutions with desired properties.

The method has the great advantage that it allows the maximum number of equally intense diffraction orders to be markedly increased. In Fig. 2, for example, where  $N_{\text{max}} = 35$  against  $N_{\text{max}} = 23$ , it is worth noting that a 35-order asymmetric grating has a higher efficiency in working orders and a wider minimum ruling  $\Delta$  than a 23-order symmetric grating.

The results obtained with all solutions, symmetric and asymmetric, are summarized in Fig. 4. It should be recognized that asymmetric gratings have a considerable advantage in such an essential fabrication parameter as the width of the minimum ruling in the period. Moreover, the tolerance value of this parameter admits a high efficiency and depth of profile, which corresponds to the phase modulation  $\phi = \pi$ . (Such gratings are known to allow for the largest deviations from the nominal depth of profile in fabrication.)

Figure 3 compares the profiles of gratings that ensure 17 equally intensive diffraction orders. The asymmetric design is clearly superior to the symmetric design in practically all respects—it has 1.25-times higher efficiency  $\eta$  and a 2.2-fold wider minimum ruling. In addition, the calculations indicate that the asymmetric version tolerates twice as large a deviation of the ruling parameters from their nominal values.

It has been found, however, that asymmetric gratings are more sensitive to deviations of profile from the rectangular shape. The perfect structure shown in Fig. 1 is practically unattainable in fabrication. The actual configuration of rulings is rather complex and is hard to reproduce exactly, but with accuracy sufficient for practical purposes it can be approximated by a trapezoidal profile (Fig. 5). The key parameter that characterizes this profile is the slope width h that is assumed to be the same for all rulings.

A non-zero h is the result of either chemical etching immediately under the photoresist mask, or erosion of the mask edges under ion etching. Quite appropriately h is referred to as the etching wedge.

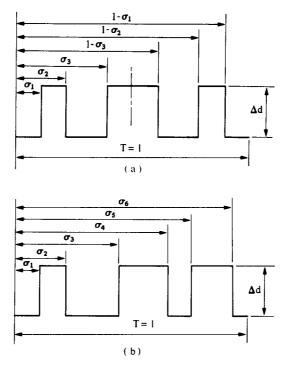


Fig. 1. Profiles of (a) symmetric and (b) asymmetric gratings with three rulings per period T.

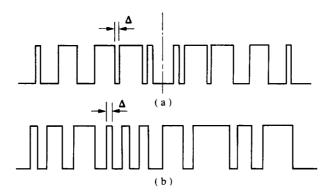


Fig. 2. Profiles of (a) symmetric (N=23,  $\eta=65\%$ ,  $\Delta=0.011$ ) and (b) asymmetric (N=35,  $\eta=75\%$ ,  $\Delta=0.020$ ) gratings with ten rulings per period.

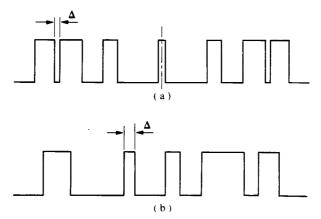


Fig. 3. Profiles of 17th-order (a) symmetric  $(K=7, \eta=61\%, \Delta=0.017, \phi=\pi)$  and (b) asymmetric  $(K=5, \eta=76\%, \Delta=0.037, \phi=\pi)$  gratings.

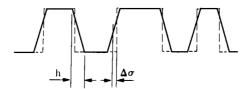


Fig. 5. Trapezoidal profile of a real grating (solid line) overlapping the perfect theoretical profile (broken line):  $\Delta \sigma$ , r.m.s. deviation of dimensions; h, etching wedge.

The deviation of the middle point of the slope from the theoretical position,  $\Delta \sigma$  (see Fig. 5) should also be taken into consideration. This quantity is assumed to have a constant magnitude for all rulings in a profile and opposite sign for adjacent slopes, namely

$$\Delta \sigma_k = (-1)^k \Delta \sigma. \tag{5}$$

This implies that there is a consistent deviation of the parameters [2] characteristic of photolithographic fabrication of gratings.

Under these conditions we consider the efficiency of the grating in the 1st diffraction order and confine ourselves to low powers of  $\Delta \sigma$  and h, for these quantities are small. We obtain the following approximate expression:

$$I'_{l} = I_{l} + \frac{4\Delta\sigma}{\pi l} \left( S_{l}C'_{l} - C_{l}S'_{l} \right) - \frac{4h}{\pi^{2}l} \left( C_{l}C'_{l} + S_{l}S'_{l} \right), \tag{6}$$

where  $I_l$ ,  $C_l$ ,  $S_l$  are given by the expressions (1) and (2), and

$$C'_{e} = \sum_{k=1}^{2K} \cos 2\pi l \sigma_{k},$$

$$C'_{l} = \sum_{k=1}^{2K} \sin 2\pi l \sigma_{k}.$$
(7)

From (6) it follows that a consistent deviation of parameters distorts the given distribution of intensities in the orders, but does not affect the symmetry of the grating spectrum, i.e. the equalities  $I_l = I_{-l}$ , for the quantity  $(S_lC'_l - C_lS'_l)/l$  does not depend on l. This effect holds for both symmetric and asymmetric gratings, the latter, as a rule, being less sensitive to the deviations of parameters (as has been already noted, a 17-order asymmetric grating tolerates about twice as large a deviation of dimensions as its symmetric counterpart).

To a first approximation, the etching wedge h does not result in scatter of intensities in different orders (to be more precise, the sums  $I_l + I_{-l}$  remain equal for different l), but it disturbs the symmetry of the spectrum

$$I_{-l} - I_l = \frac{8h}{\pi^2 |I|} \left( C_l C_l' + S_l S_l' \right). \tag{8}$$

For the symmetric period, when condition (4) holds,  $C_l = S'_l = 0$ , that is, spectral asymmetry cannot occur in principle.

Thus, an asymmetric periodic profile requires more accuracy in fabrication of rectangular rulings. On the other hand, analysis of the intensity distribution over the diffraction orders of a real diffraction grating allows the etching wedge h to be determined irrespective of parameter deviation. For a symmetrical grating this can be done only indirectly assuming a direct relation between the etching wedge and the shift of the ruling edge [2] and knowing the dimensions of rulings in the mask prior to etching.

Figure 6 presents the spectra (working orders) of two 17-order gratings made as a relief on a glass substrate. For comparison it also shows the closest theoretical spectra calculated by expressions (6)–(7). The optimal values of  $\Delta \sigma$  and h were determined by the minimum mean square deviation  $\Delta I$  of intensities in side orders of the theoretical spectrum from the real distribution. In both cases, shown in Fig. 6,  $\Delta I$  remained within 1.7%. These results prove that the trapezoidal-profile model is acceptable when the required tolerances on the profile dimensions and shape of multiple-order gratings are being evaluated.

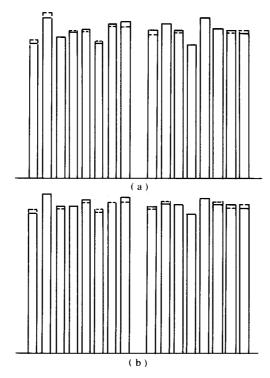


Fig. 6. Spectra of two 17-order gratings of period  $T=159~\mu m$  (solid line) and closest theoretical spectra (broken line): (a)  $\Delta \sigma = -0.18~\mu m$ ,  $h=0.55~\mu m$ ,  $\Delta I=1.7\%$ , (b)  $\Delta \sigma = -0.06~\mu m$ ,  $h=0.40~\mu m$ ,  $\Delta I=1.4\%$ .

## **REFERENCES**

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- 2. S. Bobrov, B. Kotletsov, V. Minakov et al. Diffraction gratings with orders of equal intensity. In: Golograficheskie systemy [Holographic Systems], pp. 123-129. Trudy NETI No. 2, Novosibirsk (1978).