SHORT COMMUNICATIONS

VORTEX-FREE LASER BEAM WITH AN ORBITAL ANGULAR MOMENTUM

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Abstract

We show that if one cylindrical lens is placed in the Gaussian beam waist and another cylindrical lens is placed at some distance from the first one and rotated by some angle, then the laser beam after the second lens has an orbital angular momentum (OAM). An explicit analytical expression for the OAM of such a beam is obtained. Depending on the inter-lens distance, the OAM can be positive, negative, or zero. Such a laser beam has no isolated intensity nulls with a singular phase and it is not an optical vortex, but has an OAM. By choosing the radius of the beam waist of the source Gaussian beam, the focal lengths of the lenses and the distance between them, it is possible to generate a vortex-free laser beam equivalent to an optical vortex with a topological charge of several hundreds.

Keywords: elliptic Gaussian beam, cylindrical lens, orbital angular momentum.

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Introduction

Laser beams with an orbital angular momentum (OAM) are intensively studied now due to their wide applicability in optical trapping for rotation of microscopic particles [1] and cold atoms [2], in phase contrast microscopy [3], in stimulated emission depletion microscopy [4], as well as in classical optical [5] and quantum [6] informatics. In most cases, light beams with the OAM are vortex laser beams with a singular phase and a helical (spiral) wavefront. Generally, such beams have a complex amplitude in the form $A(r)\exp(in\varphi)$, where (r, φ) are the polar coordinates, $A(r)$ is the radial term of the beam complex amplitude and n is the topological charge of the optical vortex. The OAM density and the total OAM per photon of such beams equals the topological charge n. Two questions arise. The first question is whether all laser beams with non-zero OAM have the phase dislocation and the helical wavefront or there are other beams with the OAM. The second question is what the maximal OAM is that can be practically obtained. The answer for the first question is positive and it can be found in [7], where the OAM has been calculated for an elliptic Gaussian beam focused by a cylindrical lens. Using a theoretical estimation, it has been shown in this work that the OAM of such beam can be equal to 10000 per photon. However, in [7] a beam with the OAM per photon equal only to 25 has been implemented in practice. We note that the idea of assignment of an OAM to a laser beam by using a cylindrical lens has been firstly introduced in [8]. It has been shown experimentally in [8] that after passing a cylindrical lens a Hermite-Gaussian beam without the OAM at certain propagation distance and at certain conditions transforms to a Laguerre-Gaussian beam with the OAM.

In works [9 – 12] there are attempts to answer the second question and to obtain as large as possible OAM value. In [9], it was proposed to increase the OAM by using an array of singular beams, whose axes lie on the surface of a hyperboloid of revolution. It is shown in [9] that the OAM of such composite beam can reach 204 per photon. Using a light modulator (resolution 1920×1080), entangled pairs of photons were detected in [10] with the OAM of ±300 per photon. Using an ultra-precision technology, a spiral phase mirror was manufactured in [11] on an aluminum substrate with diameter of 75 mm and roughness of 3 nm. This mirror can generate optical vortices with the topological charge of 1020. Using a spiral aluminum mirror with a diameter of about 50 mm, entangled photons for the wavelength of 810 nm were detected in [12]. The photons were entangled by the OAM and polarization, and what is more, the quantum OAM of photons was equal to ±10010. This is the maximal value of the OAM, obtained so far.

In this paper, in contrast to [7], we consider a vortex-free elliptical laser beam generated from a conventional Gaussian beam by using two cylindrical lenses, which are crossed (but not at right angle) and separated by a certain distance. An exact expression for the normalized orbital angular momentum is obtained for such beam.

1. Vortex-free beam with the OAM

In this section, first four Equations coincide with [7], for the ease of reading. Laser beams with the orbital angular momentum are usually studied within the paraxial approximation. Such beams have singular points, i.e. isolated intensity nulls with undetermined phase. Around the singular points the wavefront has a spiral shape. However, it turns out that simple light fields exist, which have the OAM, but are free of isolated intensity nulls with the helical phase. We consider an elliptic Gaussian beam after passing a cylindrical lens [7], which is placed into the beam waist and rotated in the transverse plane by an angle α. Complex amplitude immediately behind the cylindrical lens reads as

$$E(x,y)=\exp\left(-\frac{x^2}{w_x^2}-\frac{y^2}{w_y^2}\right)\times \\ \times\exp\left(-\frac{ikx^2\cos^2\alpha}{2f}-\frac{iky^2\sin^2\alpha}{2f}-\frac{ikxy\sin 2\alpha}{2f}\right), \quad (1)$$

where w_x and w_y are the waist radii of the Gaussian beam along the Cartesian axes, f is the focal length of a thin cylindrical lens, whose axis is rotated counter-clockwise by some angle α from the vertical axis y, k is the wavenumber. The normalized OAM in the paraxial case is determined by the following expressions [7] (up to constants):

$$J_z=\operatorname{Im}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{E}(x,y)\left(x\frac{\partial E(x,y)}{\partial y}-y\frac{\partial E(x,y)}{\partial x}\right)\mathrm{d}x\,\mathrm{d}y, \quad (2)$$

$$W=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{E}(x,y)E(x,y)\,\mathrm{d}x\,\mathrm{d}y, \quad (3)$$

where J_z is the axial projection of the OAM vector, W is the density of the energy (power) of light, Im is the imaginary part of a complex number, $\bar{E}$ is the complex conjugation of the amplitude (1). Substituting (1) into (2) and (3), we get a simple expression for the normalized OAM of the light field (1):

$$\frac{J_z}{W}=\left(\frac{k\sin 2\alpha}{8f}\right)\left(w_y^2-w_x^2\right). \quad (4)$$

It is seen in Eq. (4) that the OAM is zero if the Gaussian beam is circular ($w_x=w_y$) or if the lens is not inclined with respect to the vertical axis ($\alpha=0$). If all other conditions are the same, then the OAM (4) is maximal for the angle 45 deg. It is also seen in Eq. (4) that the OAM of the beam (1) is generally fractional, although it can be integer. The less is the lens focal distance and the larger is the ellipticity of the beam (1), the greater is the OAM. The OAM sign is determined by the axis along which the Gaussian beam is stretched in its waist. It can also be shown that adding an elliptic wavefront to the Gaussian beam does not affect the normalized OAM, i.e. if the complex amplitude (1) is multiplied by $\exp(ia_xx^2+ia_yy^2)$ (a_x and a_y are some real numbers), then the normalized OAM of such beam is still defined by Eq. (4). This means that the normalized OAM grows with the area of the elliptic light spot (which depends on w_x and w_y). The advantage of the beam (1) is that it can be generated without additional elements, without the light modulator, without spiral phase plate or a fork hologram. Only two cylindrical lenses are needed for its generation, one of which generates an elliptic Gaussian beam, while the other generates the OAM.

Now we estimate the OAM for specific values of variables in Eq. (4). Gaussian beam can be treated as paraxial if its waist radii exceed the wavelength. Let these radii be equal $w_x=2$ mm and $w_y=1$ mm, with the focal length being $f=10$ mm, the wavelength $\lambda=0.5$ µm, and the lens inclination angle 45 degrees ($\alpha=\pi/4$). Then the OAM in Eq. (4) equals 471.24.

Below, in difference with [7], we show that the elliptic Gaussian beam is rotating after passing the cylindrical lens. Let's derive equations to describe propagation of the beam (1) and show that no isolated intensity nulls appear on propagation, i.e. the beam (1) is not a vortex or a singular beam [9–12]. The Fresnel transform of the complex amplitude (1) reads as

$$E(\xi,\eta,z)=\frac{-ik}{z\sqrt{p(z)q(z)}}\times \\ \times\exp\left[A(z)\xi^2+B(z)\eta^2+C(z)\xi\eta\right], \quad (5)$$

where

$$A(z)=\frac{ik}{2z}-\frac{k^2}{4z^2p(z)}+\frac{k^4\sin^2 2\alpha}{64f^2z^2p^2(z)q(z)},$$
$$B(z)=\frac{ik}{2z}-\frac{k^2}{4z^2q(z)},\quad C(z)=\frac{ik^3\sin 2\alpha}{8fz^2p(z)q(z)},$$
$$p(z)=\frac{1}{w_x^2}+\frac{ik}{2z_x},\quad q(z)=\frac{1}{w_y^2}+\frac{ik}{2z_y}+\frac{k^2\sin^2 2\alpha}{16f^2p(z)}, \quad (6)$$
$$z_x=\frac{zf}{z\cos^2\alpha-f},\quad z_y=\frac{zf}{z\sin^2\alpha-f}.$$

It is seen in Eq. (5) that the Gaussian beam (1) preserves its Gaussian shape on propagation, but changes its scale and rotates. Eq. (5) is simplified significantly for $\alpha=\pi/4$ and $z=2f$, since $z_x\to\infty$ and $z_y\to\infty$ for these values:

$$E(\xi,\eta,z=2f)=-2i\gamma^{-1}\times \\ \times\exp\left[\frac{ik}{4f}\left(\xi^2+\eta^2\right)-\frac{\xi^2}{w_y^2\gamma^2}-\frac{\eta^2}{w_x^2\gamma^2}+\frac{ik\xi\eta}{2f\gamma^2}\right], \quad (7)$$

where

$$\gamma=\left(1+\frac{16f^2}{k^2w_x^2w_y^2}\right)^{1/2}. \quad (8)$$

It is seen in Eq. (7) that at the distance $z=2f$ the elliptic Gaussian beam (1) is rotated by 90 degrees and widened since $\gamma>1$.

2. Generation of an elliptic Gaussian beam

In [7], an elliptic Gaussian beam has been generated by using two cylindrical lenses. However, one cylindrical lens is sufficient for generation of converging or diverging elliptic Gaussian beam. Now we consider this in detail. Let a cylindrical lens with the curvature along the x-axis and with focal length f_1 be placed into the waist of a conventional circular Gaussian beam with the waist radius w. Then the complex amplitude of the elliptic Gaussian beam at a distance z behind the cylindrical lens reads as

$$E(x,y,z)= \\ =\frac{1}{\sqrt{q_0(z)q_1(z)}}\exp\left(-\frac{x^2}{w^2q_2(z)}-\frac{y^2}{w^2q_0(z)}\right), \quad (9)$$

where

$$q_0(z)=1+iz/z_0\ ,\quad q_1(z)=q_0(z)-z/f_1\ ,$$
$$q_2(z)=q_1(z)\left(1+iz_0\ /\ f_1\right)^{-1}\ ,\quad z_0=kw^2/2. \tag{10}$$

If a cylindrical lens with the focal length f is placed into the light field (9) and rotated by an angle α, then the complex amplitude immediately behind the lens is

$$E(x,y,z)=$$
$$=\left(q_0(z)q_1(z)\right)^{-1/2}\exp\left(-\frac{x^2}{w^2q_2(z)}-\frac{y^2}{w^2q_0(z)}\right)\times \tag{11}$$
$$\times\exp\left(-\frac{ikx^2\cos^2\alpha}{2f}-\frac{iky^2\sin^2\alpha}{2f}-\frac{ikxy\sin 2\alpha}{2f}\right).$$

The normalized OAM of the beam (11) reads as

$$\frac{J_z}{W}=\left(\frac{kw^2\sin 2\alpha}{8f}\right)\left[\frac{\left|q_0\right|^2}{\mathrm{Re}\left(q_0\right)}-\frac{\left|q_2\right|^2}{\mathrm{Re}\left(q_2\right)}\right]=$$
$$=\left(\frac{kw^2\sin 2\alpha}{8f}\right)\left(\frac{z}{f_1}\right)\left(2-\frac{z}{f_1}\right). \tag{12}$$

It is seen in Eq. (12) that the OAM tends to zero at $f_1\to\infty$, since the beam (11) tends to the conventional Gaussian beam. It is also seen in Eq. (12) that increasing of the distance z between the first cylindrical lens with the focal length f_1 and the second cylindrical lens with the focal length f allows unlimited increasing of the OAM of a laser beam. At $z=0$ and $z=2f_1$ the OAM (4) is also equal to zero, since the Gaussian beam has a circular shape. The OAM (12) has maximal positive value at $z=f_1$, i.e. when the second cylindrical lens is placed in the focus of the first one. Then, instead of Eq. (12) we get ($z=f_1$)

$$\frac{J_z}{W}=\frac{kw^2\sin 2\alpha}{8f}. \tag{13}$$

For $w_y=w$ and $w_x=0$, Eq. (13) coincides with the expression (4) for the OAM. At $z>2f_1$ the OAM (12) changes its sign (becomes negative) and is increasing (in modulus) with increasing distance z. This growth of the normalized OAM is due to the widening of the elliptic Gaussian beam during the propagation from the first cylindrical lens to the second one. The area of the light spot increases and, as we concluded above, it makes the normalized OAM increase as well.

3. Numerical results

Using the Fresnel transform, intensity and phase distributions of the field (1) were computed for several propagation distances from the cylindrical lens. The following parameters were used: wavelength $\lambda=532$ nm, Gaussian beam waist radii $w_x=20\lambda$ and $w_y=10\lambda$, cylindrical lens focal length $f=100\lambda$, inclination angle of the lens from the Cartesian coordinates $\alpha=\pi/4$, computation area $-R\le x,\ y\le R$. Normalized OAM density of the field (1) was computed by using the expression:

$$j_z=kI(x,y)(2f)^{-1}\left(y^2-x^2\right).$$

Fig. 1 shows distributions of intensity, phase and OAM density of the elliptical Gaussian beam (1) for the different distances after the cylindrical lens. It is seen in Fig. 1 that the elliptic Gaussian beam rotates on propagation after passing the cylindrical lens. The OAM density rotates with the beam synchronously, while the total OAM is certainly preserved and is equal to $J_z/W=3\pi/4$. It is also seen that at the double focal distance the Gaussian beam is turned by 90 degrees (Fig. 1*c*, left column) with respect to its initial position (Fig. 1*a*), as predicted by Eq. (7).

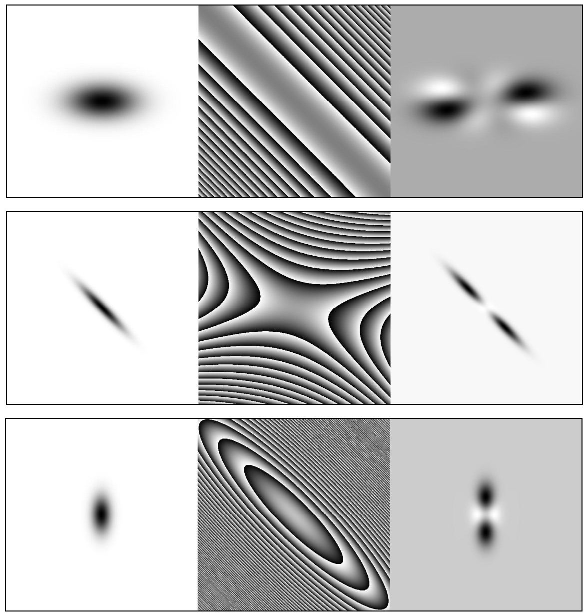

Fig. 1. Distributions of intensity (left column), phase (middle column) and OAM density (right column) of the field (1) at different distances from the initial plane: (a) z = λ (R = 40λ); (b) z = f (R = 40λ); (c) z = 2f (R = 80λ)

Conclusion

The following results are obtained in the paper. An explicit analytical expression is derived for the orbital angular momentum of a vortex-free laser beam, generated from a conventional Gaussian beam by using two cylindrical lenses, which are located at some distance from each other and which are rotated with respect to each other by some angle. This result generalizes an earlier obtained result for the beam, generated by one cylindrical lens from an initially elliptical Gaussian beam [7]. In contrast to [7], in this work explicit expressions are derived for the complex amplitude of a elliptic Gaussian beam generated by a cylindrical lens. Such beam is shown to rotate on propagation, while remaining being an elliptic Gaussian beam. At the double focal length, intensity distribution is rotated by 90 degrees.

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