Thermal entanglement in the three-qubit Tavis-Cummings model with Kerr nonlinearity

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Abstract

We obtained a solution of the Liouville equation for a system of three identical qubits interacting without detuning with a thermal field of a cavity with a Kerr medium, for two type of initial pure genuine W-entangled qubits states. On its basis, three entanglement parameters: fidelity, tangle and pairwise negativities were calculated. We obtained that sudden death of entanglement takes place for large intensities of the cavity thermal noise for all considered W-states. We also showed that Kerr nonlinearity prevents the destruction of the initial qubits entanglement induced by the thermal noise of the cavity and eliminates the sudden death of entanglement.

Keywords: qubits, cavity, thermal field, entanglement, fidelity, tangle, negativity, Kerr nonlinearity, sudden death of entanglement.

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Introduction

Multiqubit entanglement is highly relevant to solve the fundamental problems in quantum physics and plays a key role in quantum computation, quantum communication and so on [1 – 3]. To implement quantum computing operations, we need to create a series of universal gates: a two-qubit gate such as controlled NOT or CNOT plus single-qubit gate. Numerous references to possible physical implementations of two-qubit gates are presented in [4]. As a universal alternative, three-qubit gates, such as controlled-controlled-not gate (Toffoli gate) or controlled-SWAP gate (Fredkin gate), are possible. Therefore, it is particularly important to investigate the entangled states of three-qubit systems. Separable, biseparable and genuine entangled GHZ- and W-type states are possible for three-qubit systems. W-states are maximally stable not only to the loss of particles, but also to the influence of noise. The entangled W-states of qubits are used in quantum information processing. Three-qubit entangled states of both types were subsequently realized experimentally for superconducting qubits as well as for ions in traps (see references in [5 – 8]).

To generate, operate and control the entangled states of qubits the electromagnetic fields of the cavities are used. The cavities used in quantum informatics systems have finite temperatures which extend from nK for the ions in magnetic traps to room temperatures for the NV-centers in diamond (see Refs. in [9, 10]). Such cavities always contain thermal photons. So, there is a wide variation in the intensities of the thermal fields of such resonators. The effect of the thermal noise of the cavity on the qubits leads to the Rabi oscillations of the entanglement parameters of the qubits. As a result, the effect of the sudden death of entanglement may occur. In our work [10], we have studied in detail the entanglement dynamics of the system of three qubits interacting with a cavity thermal field of the resonator for separable, biseparable and genuine entangled W-type states. The negativity of the pairs of qubits in the three-qubit system was chosen as an the entanglement parameter. It would be interesting to generalize this investigation to a three-qubit system in the cavity with a nonlinear Kerr medium.

The Kerr media are widely used in a nonlinear quantum optics for squeezed states generation [11], ultrashort pulses creation [12] etc. For average phase shift of light beam in nonlinear Kerr media per photon we have $\Delta_{Kerr}=\Gamma/\varsigma$, where Γ is the Kerr constant and ς is the photon decay rate. Typically, for atomic systems in a cavity the condition $\kappa > Z$ takes place. In such a situation, the Kerr environment has little effect on atomic behavior. But recently, the ratio $Z/\kappa > 30$ has been achived for superconducting circuit (transmon) inserted in a microwave cavity [13]. This ratio is sufficient to produce the single-photon regime. Earlier, we investigated the influence of Kerr nonlinearity in the two-qubit entanglement induced by a thermal noise of infinite-Q cavity (see Refs. in [14]). We established that Kerr media can greatly enhance the amount of entanglement and eliminate the phenomenon of the sudden death of entanglement [15 – 18]. It would be interesting to generalize these results to model consisting of three qubits in a cavity.

In the present work, we investigate the influence of the Kerr media on the entanglement of the system of three identical qubits interacting with the single-mode thermal field of the lossless cavity for the initial W-type entangled states. We studied the dynamical behavior of three possible measures of the qubits entanglement. We performed a comparative analysis of the behavior of negativity of pairs of qubits, tangle and fidelity.

1. Model

Let we have three identical qubits Q_1, Q_2 and Q_3 resonantly interacting with the single-mode field of a lossless cavity. The interaction Hamiltonian of the consider model in the standard approximations has the following form

$$\mathrm{H}=\sum_{k=1}^{3}\hbar\gamma\left(\sigma_{k}^{+}a+\sigma_{k}^{-}a^{+}\right)+\hbar\Gamma a^{+2}a^{2}, \quad (1)$$

where σ_k^+ and σ_k^- are the rasing and the lowering operators in the k-th qubit , a^+ and a are the creation and annihilation operators of the cavity photons, γ is the qubit-field coupling and Γ is the Kerr nonlinearity.

Let the initial states of qubits be the W-type genuine entangled states such as

$$|\mathrm{X}_{TQ}\rangle_1 = x_1\left|1,1,2\right\rangle + y_1\left|1,2,1\right\rangle + z_1\left|2,1,1\right\rangle \quad (2)$$

with $|x_1|^2+|y_1|^2+|z_1|^2=1$ or

$$|\mathrm{X}_{TQ}\rangle_2 = x_2\left|2,2,1\right\rangle + y_2\left|2,1,2\right\rangle + z_2\left|1,2,2\right\rangle \quad (3)$$

with $|x_2|^2+|y_2|^2+|z_2|^2=1$. Here $|1\rangle_k$ is the ground state and $|2\rangle_k$ is the excited state of k-th qubit ($k=1, 2, 3$).

To generate the W-type entangled state one can use the different sets of universal gates. A natural way to implement these is to apply a resonant microwave pulses with definite durations and use the XY, ZZ or Heisenberg type interaction between qubits. For instance, M. Neeley and coauthors [5] used the so-called iSWAP universal gate to produce three-qubit entangled state of the form (2) for superconducting phase qubits. The iSWAP gate is a perfect entangling version of the SWAP gate. The authors of the paper [5] described the scheme of the three-qubit entangled state (2) generation in the following manner. The state (2) is a superposition of three states each with one qubit excited. Thus, generating the state requires “sharing” a single excitation symmetrically among three qubits. This is done by first applying a π-pulse to qubit Q_2 to excite it with one photon and create the state $|1,2,3\rangle$. To realize this operation and subsequent operations each qubit is controlled individually by a flux bias line that provides quasi-d.c. pulses for tuning the qubits in and out of resonance and a.c. (microwave) control signals for qubit rotations. In addition, each qubit is coupled to a superconducting quantum interference device (SQUID) for read-out of the qubit state. Then the qubits are entangled by turning on an equal interaction between all pairs for time $t_W=(4/9)\ t_{iSWAP}$, where $t_{iSWAP}=2\pi/\gamma$ is the time needed to complete an iSWAP gate between two qubits. To achieve symmetric interaction between all pairs of qubits the capacitive coupling network is used. This interaction described by XY Hamiltonian of the form

$$H=\mathrm{H}^{12}+\mathrm{H}^{13}+\mathrm{H}^{23},$$

where $\mathrm{H}^{ij}=\hbar\gamma/2\left(\sigma_x^i\sigma_x^j+\sigma_y^i\sigma_y^j\right)$. The interaction distribute the excitation among the qubits. A final Z-rotation can then be applied to correct the phase of qubit Q_2 .

C.F. Roos and coauthor experimentally produced the three-qubit W-state of the form (2) using an elementary ion-trap quantum processor [6]. For the investigation of tripartite entanglement, they trap three ^{40}Ca ions in a linear Paul trap. Qubits are encoded in a superposition of the $S_{1/2}$ ground state and the metastable $D_{5/2}$ state. Each ion-qubit is individually manipulated by a series of laser pulses on the quadrupole transition, with the use of narrow-band laser radiation tightly focused onto individual ions in the string. The ions electronic qubit states are initialized in the S state by optical pumping.

S. Dogra and coauthor experimentally produced three-qubit W-type entangled states of the form (2) using on a nuclear magnetic resonance (NMR) quantum-information processor [7]. The three-qubit system that be used for NMR quantum-information processing is the molecule trifluoroiodoethylene dissolved in deuterated acetone. The three qubits were encoded using the ^{19}F nuclei. The interaction between spins was of ZZ-type. W-states can be constructed by the sequence operations: one-qubit π-rotation and two CNOT and two CROT (controlled-rotation) gates.

K. R. K. Rao and A. Kumar explored the NMR quantum-information processor to a experimentally produced of the three-qubit W-type entangled states of the form (3) [8]. They used a 3-spin linear chain with nearest-neighbor Heisenberg-XY interactions The spin system chosen for the experimental implementation is ^{13}C labeled dibromo-fluoro-methane ($^{13}CHFBr_2$), where ^1H, ^{13}C and ^{19}F act as three qubits. The generation of the W-state of the form (3) was achieved by exposing the qubits to a specific sequence of NMR pulses.

The initial state of the cavity field is a thermal with a density matrix

$$\eta_F=\sum_p w_p\left|p\right\rangle\left\langle p\right|, \quad (4)$$

where

$$w_p=\langle p\rangle^p/\left(1+\langle p\rangle\right)^{p+1}$$

and

$$\langle p\rangle=\frac{1}{\exp((\hbar\Omega)/(k_BT))-1},$$

the mean number of photons of the resonator, Ω is the cavity mode frequency and T is the cavity temperature.

2. Solution of the evolution equation and entanglement criteria calculations

The whole density matrix of the considered system with Hamiltonian (1) obeys the quantum Liouville equation

$$i\hbar\frac{\partial\eta_{Q_1,Q_2,Q_3,F}}{\partial t}=\left[H,\eta_{Q_1,Q_2,Q_3,F}\right] \quad (5)$$

with initial condition

$$\eta_{Q_1,Q_2,Q_3,F}=|\mathrm{X}_{TQ}\rangle_{ii}\langle\mathrm{X}_{TQ}|\otimes\eta_F\left(i=1,2\right).$$

Before considering the interaction between qubits and thermal field, we study two qubits simultaneously interacting with one-mode Fock state. For this aim we will find a solution of the quantum Schrödinger equation for time-dependent vector $|\Psi_{Q_1Q_2Q_3F}(t)\rangle$

$$i\hbar\frac{\partial|\Psi_{Q_1Q_2Q_3F}(t)\rangle}{\partial t}=H|\Psi_{Q_1Q_2Q_3F}(t)\rangle \quad (6)$$

with initial condition

$$\left|\Psi_{Q_1Q_2Q_3F}(t)\rangle\right|_{t=0}=|\mathrm{X}_{TQ}\rangle_i\otimes|n\rangle\,(i=1,2).$$

To obtain the solution of equation (6) we used the so-called "dressed" states representation. Suppose that the excitation number of the atom-field system is $p\,(p\geq 0)$. The evolution of the system is confined in the subspace

$$|1,1,1;p+3\rangle,|2,1,1;p+2\rangle,|1,2,1;p+2\rangle,$$
$$|1,1,2;p+2\rangle,|2,2,1;p+1\rangle,|2,1,2;p+1\rangle,$$
$$|1,2,2;p+1\rangle,|2,2,2;p\rangle.$$

Thus, the "dressed" states or eigenfunctions of the Hamiltonian (1) can be written as

$$|\Phi_{i,p}=\xi_{i,p}(X_{i1,p}|1,1,1;p+3\rangle+X_{i2,p}|W_1;p+2\rangle+$$
$$+X_{i3,p}|W_2;p+1\rangle+X_{i4,p}|2,2,2;p\rangle),$$

where

$$|W_1;p+2\rangle=(1/\sqrt{3})(|2,1,1;p+2\rangle+$$
$$|1,2,1;p+2\rangle+|1,2,1;p+2\rangle),$$
$$|W_2;p+1\rangle=(1/\sqrt{3})(|2,2,1;p+1\rangle+$$
$$|2,1,2;p+1\rangle+|1,2,2;p+1\rangle),$$

and

$$\xi_{i,p}=\frac{1}{\sqrt{\left|X_{i1,p}\right|^2+\left|X_{i2,p}\right|^2+\left|X_{i3,p}\right|^2+\left|X_{i4,p}\right|^2}}.$$

The coefficients $X_{ij,p}$ obeys the equations

$$\begin{aligned}&\chi p(p-1)X_{1i,p}+3\sqrt{p+1}X_{2i,p}=\varepsilon_{i,p}X_{1i,p},\\&\sqrt{3(p+1)}X_{1i,p}+3\chi(p+1)pX_{2i,p}+\\&+2\sqrt{3(p+2)}X_{3i,p}=\varepsilon_{i,p}X_{2i,p},\\&\sqrt{12(p+2)}X_{2i,p}+\sqrt{3}\chi(p+2)(p+1)X_{3i,p}+\\&+\sqrt{3(p+3)}X_{4i,p}=\varepsilon_{i,p}X_{3i,p},\\&3\sqrt{p+3}X_{2i,p}+\chi(p+3)(p+2)X_{4i,p}=\varepsilon_{i,p}X_{4i,p},\end{aligned}\quad (7)$$

where $\chi=\Gamma/\gamma, \varepsilon_{i,p}=E_{i,p}/\hbar\gamma$. For model with Kerr nonlinearity the solution of equations (7) can only be found by numerical calculations. Here we present these for model without Kerr nonlinearity:

$$X_{11,p}=\sqrt{\frac{(A_p-B_p)(C_p-\sqrt{3}B_p)}{6\sqrt{(p+1)(p+2)(p+3)}}},$$

$$X_{12,p}=\sqrt{\frac{(D_p+\sqrt{3}B_p)(C_p-\sqrt{3}B_p)}{6\sqrt{(p+2)(p+3)}}},$$

$$X_{11,p}=-\sqrt{\frac{(A_p-B_p)}{\sqrt{3(p+3)}}},\ X_{14,p}=1,$$

$$X_{21,p}=-X_{11,p},X_{22,p}=X_{12,p},\ X_{23,p}=-X_{13,p},$$
$$X_{24,p}=X_{14,p},\ X_{31,p}=X_{21,p},\ X_{32,p}=X_{22,p},$$
$$X_{33,p}=X_{23,p},\ X_{34,p}=X_{24,p},\ X_{41,p}=-X_{11,p},$$
$$X_{42,p}=X_{12,p},\ X_{43,p}=-X_{13,p},\ X_{44,p}=X_{14,p},$$

where

$$A_p=4+\sqrt{3}(p+2)+2p,$$
$$B_p=19+16\left(\sqrt{3}+\sqrt{3}p+p\right)+4p^2\left(\sqrt{3}+1\right),$$
$$C_p=3+2\sqrt{3}(p+2),\ D_p=3-2\sqrt{3}(p+2).$$

The corresponding eigenvalues are

$$E_{1,p}=-\sqrt{3}\hbar\gamma\sqrt{A_p-B_p},\ E_{2,p}=\sqrt{3}\hbar\gamma\sqrt{A_p-B_p},$$
$$E_{3,p}=-\sqrt{3}\hbar\gamma\sqrt{A_p+B_p},\ E_{4,p}=\sqrt{3}\hbar\gamma\sqrt{A_p+B_p}.$$

Assume that the considered system is initially in the state $|\Psi\rangle_{TQ,1}\otimes|p\rangle$ with $x_1=y_1=z_1=1/\sqrt{3}$, then at time t, the whole system will evolve to

$$|\Psi_{Q_1Q_2Q_3F}(t)\rangle_p=V_{12,p-2}|-,-,-;p+1\rangle+$$
$$+V_{22,p-2}|W_1;p\rangle+V_{32,p-2}|W_2;p-1\rangle+$$
$$+V_{42,p-2}|+,+,+;p-2\rangle,$$

where

$$\begin{aligned}V_{i2,p}&=\exp(-iE_{1,p}t/\hbar)\xi_{1,p}Y_{2i,p}X_{1i,p}+\\&+\exp(-iE_{2,p}t/\hbar)\xi_{2,p}Y_{2i,p}X_{2i,p}+\\&+\exp(-iE_{3p}t/\hbar)\xi_{3,p}Y_{2i,p}X_{3i,p}+\\&+\exp(-iE_{4,p}t/\hbar)\xi_{4,p}Y_{2i,p}X_{4i,p},\end{aligned}\quad (8)$$

and

$$Y_{ij,p}=\xi_{j,p}X^{*}_{ji,p}\quad(i=1,2,3,4).$$

If the initial state is $|\Psi\rangle_{TQ,2}\otimes|p\rangle$ with $x_2=y_2=z_2=1/\sqrt{3}$ the time-dependent state vector takes the form

$$\begin{aligned}|\Psi_{Q_1Q_2Q_3F}(t)\rangle_p&=V_{13,p-1}|-,-,-;p+2\rangle+\\&+V_{23,p-1}|W_1;p+1\rangle+V_{33,p-1}|W_2;p\rangle+\\&+V_{43,p-1}|+,+,+;p-1\rangle,\end{aligned}\quad (9)$$

where the coefficients $V_{i3,p}$ can be obtained from (8) by replacing $Y_{2i,p}$ with $Y_{3i,p}$ $(i=1,2,3,4)$.

Using the formulas (8) or (9) we can biuld the solution of the Liouville equation (5) as

$$\eta_{Q_1,Q_2,Q_3,F}(t)=\sum_p w_p\,|\Psi_{Q_1Q_2Q_3F}(t)\rangle_{pp}\langle\Psi_{Q_1Q_2Q_3F}(t)|.$$

Using these solutions for whole density matrixes $\eta_{Q1,Q2,Q3,F}$ one can calculate the three-qubit reduced matrixes

$$\eta_{Q_1,Q_2,Q_3}=Tr_F\,\eta_{Q_1,Q_2,Q_3,F}, \quad (10)$$

the pairwise reduced two-qubit density matrixes

$$\eta_{Q_i,Q_j}=Tr_{Q_k}\eta_{Q_1,Q_2,Q_3}\left(i\neq j\neq k; i,j,k=1,2,3\right), \quad (11)$$

and one-qubit reduced density matrixes following form.

$$\eta_{Q_i}=Tr_{Q_j}\eta_{Q_i,Q_j}\left(i\neq j, i,j=1,2,3\right). \quad (12)$$

Using the formulas (10)–(12) one can obtain all known entanglement criteria for qubits subsystem. The modern review of the theory of multipartite entanglement measures has been presented in [19].

In this paper, we concern our attention on calculation of three measures of multipartite entanglement that are most widely used in quantum informatics. The first of them is the fidelity. This parameter shows the degree of coincidence between time-dependent three-qubit state and initial state. In the case of a thermal field of a cavity, the state of the qubits at an arbitrary moment of time is mixed. The fidelity for mixed states of qubits was proposed in [20]

$$\Phi(\eta,\eta')=\left(tr\sqrt{\eta^{1/2}\eta'\eta^{1/2}}\right)^2,$$

where η is the initial three-qubit density matrix and η' is the three-qubit density matrix for $t>0$. When the initial density matrix η describe the pure state we have $\sqrt{\eta}=\eta$. For initial pure qubits state (2) and (3) the initial density matrixs are $\eta=|X_{TQ}\rangle_{ii}\langle_{TQ}X|\,(i=1,2)$. In this case the fidelity can be writing in the form

$$\Phi(t)=\left(tr\sqrt{|\,\mathrm{X}_{TQ}\rangle_{ii}\langle_{TQ}\mathrm{X}|\eta_{Q_1,Q_2,Q_3}|\mathrm{X}_{TQ}\rangle_{ii}\langle_{TQ}\mathrm{X}\,|}\right)^2=$$
$$={}_i\langle_{TQ}\mathrm{X}|\eta_{Q_1,Q_2,Q_3}|\mathrm{X}_{TQ}\rangle_i=tr(|\,\mathrm{X}_{TQ}\rangle_{ii}\langle_{TQ}\mathrm{X}\,|\,\eta_{Q_1,Q_2,Q_3}).$$

For initial pure entangled qubits state (2) and (3) fidelity $\Phi(0)=1$.

As the second criterion of the qubit entanglement we used the tangle [21]. For thee-qubit system tangle takes the form

$$\mathrm{T}(t)=\mathrm{T}_{Q_1}(t)-\mathrm{T}_{Q_1,Q_2}(t)-\mathrm{T}_{Q_1,Q_3}(t),$$

where $\mathrm{T}_{Q_1}(t)=4det|\eta_{Q_1}|, \mathrm{T}_{Q_i,Q_j}=S_{ij}^2$ and S_{ij} is the concurrence (Wootters criterion) between qubits i and $j\,(i\neq j, i,j,=Q_1,Q_2,Q_3)$ [22]. The two-qubit concurrence defined by a standard manner as

$$S_{ij}(t)=max\left\{\sqrt{\mu_1(t)}-\sqrt{\mu_2(t)}-\sqrt{\mu_3(t)}-\sqrt{\mu_4(t)},0\right\},$$

where μ_1,μ_2,μ_3,μ_4 are the eigenvalues, in decreasing order, of the Hermitian matrix

$$\sqrt{\sqrt{\eta_{Q_i,Q_j}}\tilde{\eta}_{Q_i,Q_j}\sqrt{\eta_{Q_i,Q_j}}}.$$

Here

$$\tilde{\eta}_{Q_i,Q_j}=\left(\sigma_y\otimes\sigma_y\right)\eta^*_{Q_i,Q_j}\left(\sigma_y\otimes\sigma_y\right),$$

and

$$\sigma_y=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

is the Pauli matrix (y-component). For initial entangled W-states (2) and (3) the tangle at the initial time $\mathrm{T}(0)=0$.

As the third entanglement criterion we used the pairwise negativity (Peres-Horodeckii criterion). Two-qubit negativities has the form [23–25]

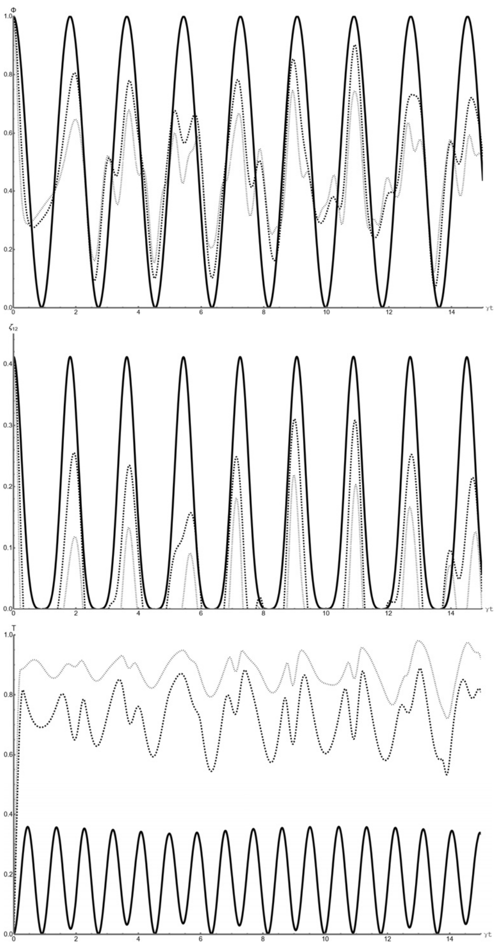
$$\zeta_{12}(t)=-2\sum_l\lambda^-_{l,12},\quad \zeta_{23}(t)=-2\sum_l\lambda^-_{l,23},$$
$$\zeta_{13}(t)=-2\sum_l\lambda^-_{l,13},$$

where $\lambda^-_{l,12}$ $\lambda^-_{l,23}$ and $\lambda^-_{l,13}$ are the negative eigenvalues of the two-qubit density matrixes partially transposed in variables of one qubit $\eta^{T_1}_{Q_1,Q_2}(t)$, $\eta^{T_1}_{Q_2,Q_3}(t)$ and $\eta^{T_1}_{Q_1,Q_3}(t)$ correspondently. For entangled qubits i and j we have $0<\zeta_{ij}\leq 1$. The value $\zeta=1$ correspond to maximal amount of entanglement between qubits i and j.

3. Results and discussion

The results of computer modeling of the fidelity, tangle and pairwise negativities for entangled qubits states (2) and (3) and thermal field are shown in figs. 1–4. Fig. 1 presented the three introduced parameters as a functions of scaled time γt. The initial entangled qubits state (2) and different values of the mean number of photons in the cavity mode assumed to be under consideration.

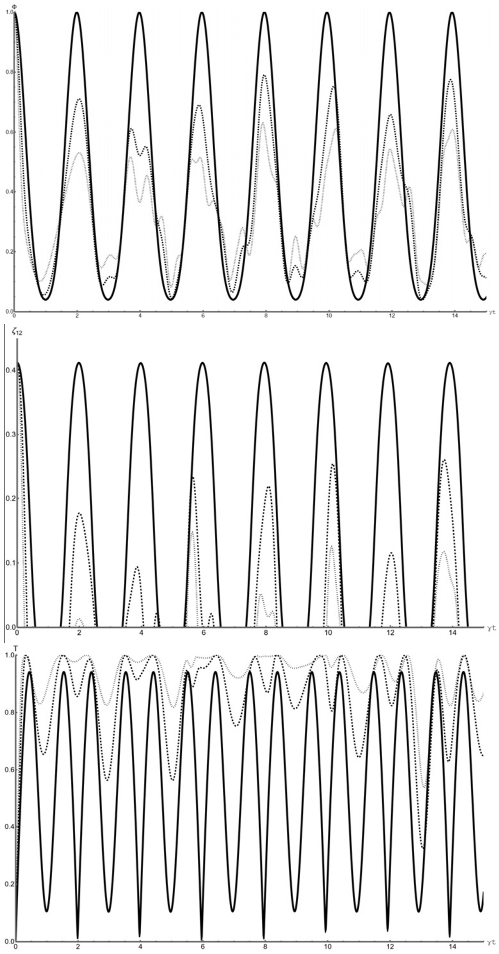
An analysis of the time behavior of all three parameters shows that with increasing thermal noise intensity, the maximum amount of entanglement of both the pairs of qubits and three qubits decreases. A comparison of the fig. 1*a* and *b* shows that behavior of the fidelity and pairwise negativity for different values of mean photon numbers. But the behavior of negativity allows us to note some features in the dynamics of qubits entanglement that are absent in the dynamics of tangle or fidelity. This demonstrates the effect of sudden death of entanglement for large thermal field intensities. Fig. 1*b* clearly shows that at some times the pairwise entanglement disappears abruptly and remains zero for a finite time before being reborn. This means there is an sudden death of entanglement. Calculations show that the effect of sudden death of entanglement is absent only for low intensity thermal fields and for the initial truly entangled state (2). It is also worth noting that the larger the mean value of the photon number, the longer is the period of time during which there is no entanglement of qubits. Fig. 2 demonstrates the same as in fig. 1 but for another initial qubits state (3). Comparing the behavior of entanglement parameters in fig. 1 and 2 we can established that maximum entanglement amount for state (3) decreases significantly faster than that state (2) as the mean number of photons increases. One can see from fig. 2*b* that, unlike the previous case, the sudden death of entanglement takes place for all intensities of the thermal field and the periods of the entanglement lack becomes much longer. The presented results allow us to conclude that the genuine three-qubit entangled states (2) are more stable with respect to thermal noise than genuine entangled states (3) and therefore are better suited for quantum information processing.



a) *b)* *c)*

Fig.1. Fidelity $\Phi(\gamma t)$ a), negativity $\zeta_{ij}(\gamma t)$ b) and tangle $T(\gamma t)$ c) as functions of rescaled time γt for model without nonlinearity and initial entangled state (2) with $x_1 = y_1 = z_1 = 1/\sqrt{3}$. The mean number of thermal photons: $\langle p \rangle = 0.001$ (solid line), $\langle p \rangle = 1$ (dashed line), $\langle p \rangle = 2.5$ (dotted line)

Fig. 3 shows the entanglement parameters as functions of rescaled time γt for initial entangled qubits state (2), fixed values of the mean photon number and different values of the Kerr nonlinearity. The time behavior of all curves demonstrate that nonlinearity greatly enhances the maximum value of entanglement induced by a thermal noise. Thus, the Kerr media stabilizes the initial qubits entanglement. Fig. 3*c* also shows that Kerr nonlinearity eliminates the effect of the sudden death of entanglement. Fig. 4 shows the same as in fig. 3 but for initial state (3). Comparing the behavior of entanglement parameters in figs. 3 and 4 one can see that the dependences of qubit entanglement parameters on the nonlinearity for states (2) and (3) are qualitatively very similar. However, comparing fig. 3*c* and fig. 4*c*, we can conclude that the phenomenon of sudden death of entanglement disappears at much smaller values of the Kerr nonlinearity for the initial state (3). Summarizing, we can note that Kerr nonlinearity can act as an effective mechanism for operating and controlling the thermal entanglement of a three-qubit systems.



a) *b)* *c)*

Fig.2. Fidelity $\Phi(\gamma t)$ a), negativity $\zeta_{ij}(\gamma t)$ b) and tangle $T(\gamma t)$ c) as functions of rescaled time γt for model without nonlinearity and the initial entangled state (3) with $x_2 = y_2 = z_2 = 1/\sqrt{3}$. The mean number of thermal photons: $\langle p \rangle = 0.001$ (solid line), $\langle p \rangle = 1$ (dashed line), $\langle p \rangle = 2.5$ (dotted line)

Conclusion

We investigated the dynamics of the system of three identical qubits resonantly interacting with a the single-mode thermal field of an infinite-Q cavity with Kerr nonlinearity. The genuine entangled W-type states of qubitswere in the focus of our attention. To generate such entangled W-type states, an NMR quantum information processor and elementary quantum processor with an ion trap, can be used. We obtained the solutions of the evolution equation for the density matrix of the system “three qubits+field mode” with Hamiltonian (1). We used this solution to calculate the of three entanglement parameters: fidelity, tangle and pairwise negativity. The results of investigations of the entanglement parameters time dependences showed that the thermal field destroys the initial entanglement of qubits and the sudden death of entanglement takes place for almost all considered initial qubits states. An exception is the case when the qubits are prepared in the initial state (2), and the field is in thermal state of low intensity. We also concluded that the genuinethree-qubit entangled states (2) are more stable with respect to thermal noise than states (3). It was also found that Kerr nonlinearity weakens the degradation of the initial qubits entanglement and eliminates the sudden death of entanglement. Summarizing, Kerr nonlinearity can be considered as an effective mechanism that prevents errors from occurring in quantum information processing.

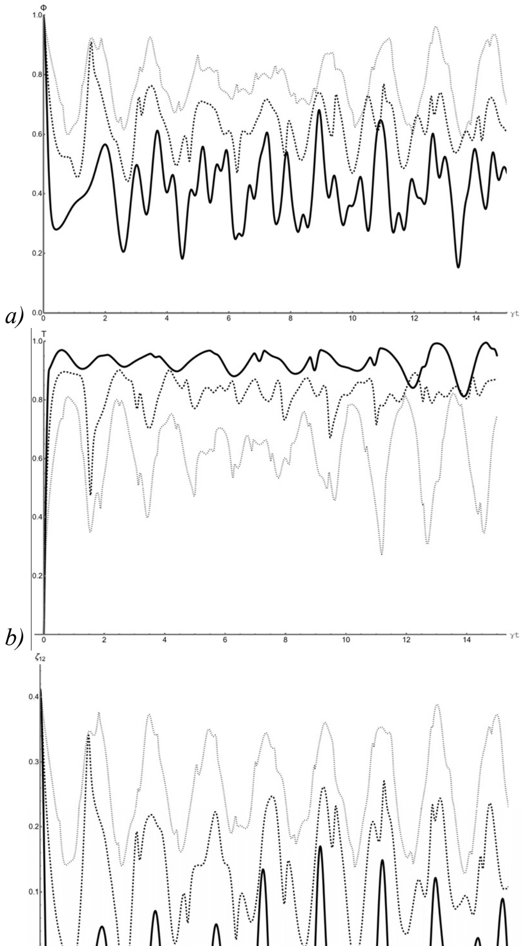


Fig.3. Fidelity $\Phi(\gamma t)$ a), tangle $T(\gamma t)$ b) and negativity $\zeta_{ij}(\gamma t)$ c) functions of rescaled time γt for model with Kerr nonlinearity and initial entangled state (2) with $x_1 = y_1 = z_1 = 1/\sqrt{3}$. The fixed value of the mean number of thermal photons $\langle p\rangle = 4$. The Kerr nonlinearity: $\Gamma = 0$ (solid line), $\Gamma = 2\gamma$ (dashed line), $\Gamma = 5\gamma$ (dotted line)

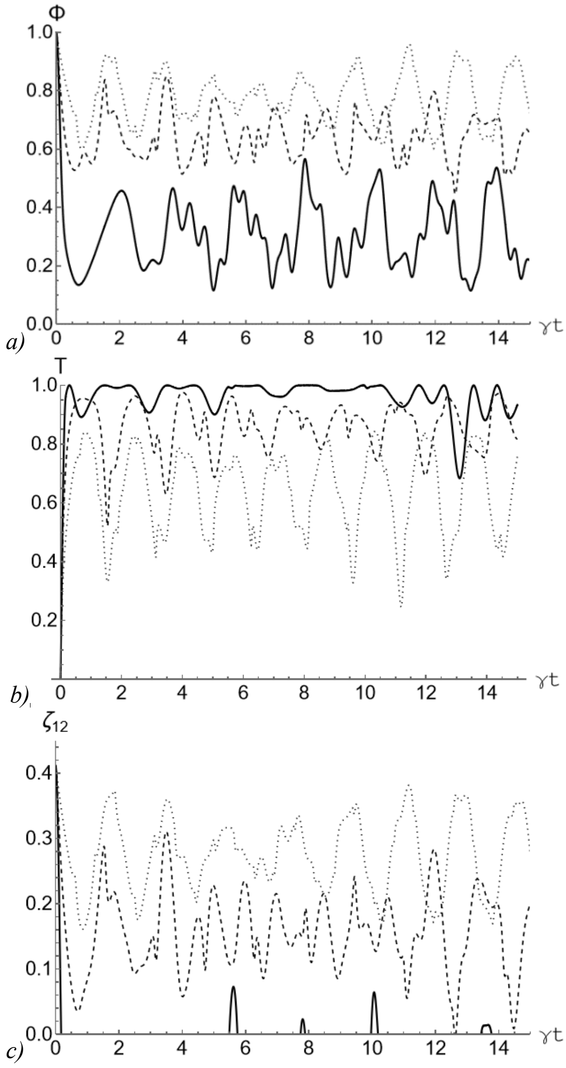


Fig.4. Fidelity $\Phi(\gamma t)$ a), tangle $T(\gamma t)$ b) and negativity $\zeta_{ij}(\gamma t)$ c) functions of rescaled time γt for model with Kerr nonlinearity and initial entangled state (3) with $x_2 = y_2 = z_2 = 1/\sqrt{3}$. The fixed value of the mean number of thermal photons $\langle p\rangle = 4$. The Kerr nonlinearity: $\Gamma = 0$ (solid line), $\Gamma = 2\gamma$ (dashed line), $\Gamma = 5\gamma$ (dotted line)

When calculating the entanglement parameters of three qubits in the framework of the considered model, we the interaction of our system with environment. The investigation of the entanglement dynamics of the considered systems with taking into account the dissipative processes in non-markovian approximation will be a subject of our next paper.

References

[1] Alexeev Y, Bacon D, Brown KR, Calderbank R, Carr LD, Chong FT, DeMarco B, Englung D, Farhi E, Fefferman B, Gorshkov AV, Houck A, Kim J, Kimmel S, Lange M, Lloyd S, Lukin MD, Maslov D, Maunz P, Monroe C, Preskill J, Roetteler M, Savage MJ, Thompson J. Quantum computer systems for scientific discovery. PRX Quantum 2021; 2(1): 017001. DOI: 10.1103/PRXQuantum.2.017001.

[2] Souza AM, Sarthour RS, Oliveria LS, Vedral V. Entanglement in many body systems. Phys Rev B Condens 2023; 653: 414511. DOI: 10.1016/j.physb.2022.414511.

[3] Duarte FJ. Fundamentals of quantum entanglement. Bristol: IOP Publishing; 2022. ISBN: 978-0750352659.
[4] Barshak EV, Lapin BP, Vikulin DV, Alieva SS, Alexeyev CN, Yavorsky MA. All-fiber SWAP-CNOT gate for optical vortices. Computer Optics 2021; 45(6): 853-859. DOI: 10.18287/2412-6179-CO-938.
[5] Neely M, ed. Generation of three-qubit entangled states using superconducting phase qubits. Nature 2010; 467(7315): 570-573. DOI: 10.1038/nature09418.
[6] Roos CF, ed. Control and measurement of three-qubit entangled states. Science 2004; 304: 1478-1480. DOI: 10.1126/science.1097522.
[7] Dogra S, Dorai K, Arvind. Experimental construction of generic three-qubit states and their reconstruction from two-party reduced states on an NMR quantum information processor. Phys Rev A 2015; 91(2): 022312. DOI: 10.1103/PhysRevA.91.022312.
[8] Rao KRK, Kumar A. Entanglement in a 3-spin Heisenberg-XY chain with nearest-neighbor interactions, simulated in an NMR quantum simulator. Int. J. Quant. Inform. 2012; 10(4): 1250039. DOI: https://doi.org/10.1142/S0219749912500396.
[9] Bashkirov EK. Thermal entanglement of two atoms with dipole-dipole and Ising interactions. Proc SPIE 2022; 12193: 121930R. DOI: 10.1117/12.2625842.
[10] Bagrov AR, Bashkirov EK. Sudden death of entanglement in a thermal three-qubit Tavis-Cummings model. Proc 9th IEEE Int Conf on Information Technology and Nanotechnology 2023: 1-5. DOI: 10.1109/ITNT57377.2023.10139206.
[11] Walls D. Squeeze states of light. Nature 1983; 306: 141-146. DOI: 10.1038/306141a0.
[12] Fenwick KL, England DG, Bustard PJ, Fraser JM, Sussman BJ. Carving out configurable ultrafast pulses from a continuous wave source via the optical Kerr effect. Opt Express 2020; 28(17): 24845-24853. DOI: 10.1364/OE.399878.
[13] Kirchmair G, Vlastakis B, Leghtas Z, Nigg SE, Paik H, Ginossar E, Mirrahimi M, Frunzio L, Girvin SM, Schoelkopf RJ. Observation of quantum state collapse and revival due to the single-photon Kerr effect. Nature 2013; 495: 205-209. DOI: 10.1038/nature11902.
[14] Bashkirov EK. Thermal entanglement in Tavis-Cummings models with Kerr media. Proc SPIE 2022; 12193: 121930Q. DOI: 10.1117/12.2625838.
[15] Yu T, Eberly JH. Sudden death of entanglement. Science 2009; 323(5914): 598-601. DOI: 10.1126/science.1167343.
[16] Decordi GL, Vidiella-Barranco A. Sudden death of entanglement induced by a minimal thermal environment. Opt Commun 2020; 47515: 126233. DOI: 10.1016/j.optcom.2020.126233.
[17] Xie S, Younis D, Eberly JH. Evidence for unexpected robustness of multipartite entanglement against sudden death from spontaneous emission. Phys Rev Res 2023; 5(3): L032015. DOI: 10.1103/PhysRevResearch.5.L032015.
[18] Awasthi N, Joshi DK. Sustainability of entanglement sudden death under the action of memory channel. Laser Phys Lett 2023; 20(2): 025202. DOI: 10.1088/1612-202X/acaece.
[19] Ma M, Li Y, Shang J. Multipartite entanglement measures: a review. arXiv Preprint. 2023. Source: <https://arxiv.org/abs/2309.09459>. DOI: 10.48550/arXiv.2309.09459.
[20] Liang Y-C, Yeh Y-H, Mendonca PEMF, The RY, Reid MD, Drummond PD. Quantum fidelity measures for mixed states. Rep Prog Phys 2019; 82(7): 076001. DOI: 10.1088/1361-6633/ab1ca4.
[21] Coffman V, Kundu J, Wootters WK. Distributed entanglement. Phys Rev A 2000; 61(5): 052306. DOI: 10.1103/PhysRevA.61.052306.
[22] Wootters WK. Entanglement of formation and concurrence. Quant Inform Comput 2001; 1(1): 27-44. DOI: 10.26421/QIC1.1-3.
[23] Schneeloch J, Tison CC, Jacinto HS, Alsing PM. Negativity vs. purity and entropy in witnessing entanglement. Sci Rep 2023; 13(1): 4601. DOI: 10.1038/s41598-023-31273-9.
[24] Horodecki R, Horodecki M, Horodecki P, Horodecki K. Quantum entanglement. Rev Mod Phys 2009; 81(2): 865-942. DOI: 10.1103/RevModPhys.81.865.
[25] Wang A-M. Eigenvalues, Peres' separability condition, and entanglement. Commun Theor Phys 2004; 42(2): 206-210. DOI: 10.1088/0253-6102/42/2/206.

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