Identification of parameters of discrete stochastic systems with unknown inputs based on an information filtering algorithm

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Abstract

The paper considers the problem of parameter identification of discrete linear stochastic systems in the state space. An identification criterion is proposed for systems with unknown inputs based on the information version of the Gillijns and De Moor algorithm. We apply this criterion to identify the diffusion coefficient of a one-dimensional diffusion model with unknown boundary conditions of the first kind. The results of computer modeling validate the presented approach.

Keywords: discrete linear stochastic systems, parameter identification, unknown exogenous inputs, information filtering, identification criterion, diffusion model.

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Introduction

The problems of estimation and identification of parameters of dynamic systems play an important role in the theory of control and signal processing. The main objective of estimation is to determine the current state of the system based on available measurements, while parameter identification is aimed at establishing the unknown parameters of the model, which may vary in time or be uncertain initially. These two tasks are closely related because accurate knowledge of the system parameters allows for more efficient estimation of the system state, and the relationship between the estimation results and the observed data can be used to refine the parameters. They become particularly important for systems with unknown inputs, which may represent external disturbances such as noise, sensor errors, or technical faults that affect the dynamics of the system. If these inputs are not considered in state estimation or parameter identification, this can lead to errors in predicting system behavior.

A number of estimation and identification methods have been developed to simultaneously consider the system state and unknown inputs. Such methods are of great practical importance in fields ranging from technical diagnostics to geophysics and medicine. In [1], a review of estimation and identification methods for systems with unknown inputs is given and an approach to solving the problem of parameter identification of discrete linear stochastic systems with unknown inputs is proposed. This approach is based on the construction and minimization of the identification criterion which is calculated using values obtained by the algorithm of simultaneous estimation of the state vector and unknown inputs proposed by S. Gillijns and B. De Moor (GDM) in [2]. This algorithm has a covariance form. It is known that in some cases, for example, when the initial covariance of the state vector is unknown, it is preferable to use information-type algorithms and their modifications to solve the estimation problem [3]. In this regard, the task of developing identification methods based on information versions of estimation algorithms is relevant. It also should be noted that there are few existing works on identification of stochastic systems with unknown exogenous inputs (see, for example, [4 – 6]).

The identification criterion proposed in this paper is a further development of the results in [1], employing the information version of the GDM algorithm.

1. Problem statement

Consider a discrete linear stochastic system

$$\begin{cases} x_k = F x_{k-1} + B d_{k-1} + G w_k, \\ z_k = H x_k + v_k, \ \ k = 1,2,\dots,K, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $d_k \in \mathbb{R}^r$ is the unknown input vector, $z_k \in \mathbb{R}^m$ is the measurement vector; matrices $F \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$, $G \in \mathbb{R}^{n\times q}$, $H \in \mathbb{R}^{m\times n}$ are constant; K is the number of measurements; the initial state $x_0 \sim \mathcal{N}(\bar{x}_0, \Pi_0)$ and additive noises $w_k \in \mathbb{R}^q \sim \mathcal{N}(0, Q)$ and $v_k \in \mathbb{R}^m \sim \mathcal{N}(0, R)$ are mutually independent. The covariance matrices Q and R are positive definite.

Assume that the matrices F, G, Q, R in (1) are known up to the parameter $\theta \in \mathbb{R}^p$:

$$\begin{cases} x_k = F(\theta) x_{k-1} + B d_{k-1} + G(\theta) w_k, \\ z_k = H x_k + v_k, \ \ k = 1,2,\dots,K. \end{cases} \tag{2}$$

It is required to identify the unknown parameter θ based on the results of observations $Z_1^K = \{z_1, \dots, z_k, \dots, z_K\}$.

2. Information filtering in the presence of unknown inputs

Assume that the following conditions are met [1, Theorem 1]:

$$\operatorname{rank} H = n, \operatorname{rank} HB = \operatorname{rank} B = r. \quad (3)$$

Each iteration of the GDM algorithm [2] consists of the following steps:
1. Time update.
2. Input estimation.
3. Measurement update.

At each iteration of the algorithm, estimates $\hat{d}_{k-1}$ and $\hat{x}_k$ of the unknown input vector and the state vector are calculated as well as the corresponding covariance matrices of estimation errors D_{k-1} and P_k. Let I_n be an identity matrix of size $n \times n$.The covariance version of the GDM algorithm can be formulated as follows:

Algorithm 1 (GDM)

for $k = 1,2, \dots, K$ **do**

Time Update

$\hat{x}_{k|k-1} = F\hat{x}_{k-1}$

$P_{k|k-1} = FP_{k-1}F^T + GQG^T$

Input Estimation

$\tilde{R}_k = HP_{k|k-1}H^T + R$

$D_{k-1} = (B^T H^T \tilde{R}_k^{-1} HB)^{-1}$

$M_k = D_{k-1}B^T H^T \tilde{R}_k^{-1} = (HB)^+$

$\hat{d}_{k-1} = M_k(z_k - H\hat{x}_{k|k-1})$

Measurement Update

$K_k = P_{k|k-1}H^T \tilde{R}_k^{-1}$

$\hat{x}_k^* = \hat{x}_{k|k-1} + B\hat{d}_{k-1}$

$P_k^* = (I_n - K_k H)P_{k|k-1}$

$\hat{x}_k = \hat{x}_k^* + K_k(z_k - H\hat{x}_k^*)$

$P_k = P_k^* + (I_n - K_k H)BD_{k-1}B^T(I_n - K_k H)^T$

end for

The information version of the GDM algorithm as well as its square-root modification are proposed in [7]. Suppose that F is invertible. Let us denote $Y_k = P_k^{-1}$, $y_k = Y_k\hat{x}_k$, $T_k = D_k^{-1}$, $t_k = T_k\hat{d}_k$, then the GDM algorithm in information form may be written as follows:

Algorithm 2 (IGDM)

for $k = 1,2, \dots, K$ **do**

Time Update

$A_{k-1} = F^{-T}Y_{k-1}F^{-1}$

$\tilde{Q}_{k-1} = G^T A_{k-1} G + Q^{-1}$

$L_{k-1} = A_{k-1}G\tilde{Q}_{k-1}^{-1}$

$S_{k-1} = I_n - L_{k-1}G^T$

$Y_{k|k-1} = S_{k-1}A_{k-1}$

$y_{k|k-1} = S_{k-1}F^{-T}y_{k-1}$

Input Estimation

$\bar{Y}_k = Y_{k|k-1} + H^T R^{-1} H$

$\bar{T}_k = B^T Y_{k|k-1} B$

$T_{k-1} = \bar{T}_k - B^T Y_{k|k-1} \bar{Y}_k^{-1} Y_{k|k-1} B$

$t_{k-1} = -B^T y_{k|k-1} + B^T Y_{k|k-1} \bar{Y}_k^{-1} (y_{k|k-1} + H^T R^{-1} z_k)$

Measurement Update

$Y_k = \bar{Y}_k - Y_{k|k-1} B \bar{T}_k^{-1} B^T Y_{k|k-1}$

$y_k = y_{k|k-1} + H^T R^{-1} z_k - Y_{k|k-1} B \bar{T}_k^{-1} B^T y_{k|k-1}$

end for

3. Construction of the identification criterion

As shown in [1], the state vector estimates produced by the GDM algorithm can be used to identify the unknown parameter θ of system (2).

Consider a process of discrete-time filtering provided by Algorithm 1. Since matrices in (2) depend on θ, then the estimation error $e_k(\theta) = x_k - \hat{x}_k(\theta)$ in the filtering equations will depend on the value of parameter θ. The minimum value of $e_k(\theta)$ can be obtained under the condition of a minimum by θ of the original identification criterion (OIC) in the form of quadratic functional

$$\mathcal{J}_e(\theta, K) = \mathbb{E}\{e_k^T(\theta) e_k(\theta)\}. \tag{4}$$

The problem is that this functional is not instrumental, that is, practically feasible, since the estimation errors $e_k(\theta)$ are not directly observable.

Let us construct an observable process $\varepsilon_k(\theta)$ in the form

$$\varepsilon_k(\theta) = W^+ z_k - \hat{x}_k^*(\theta) \tag{5}$$

where $W^+ = (H^T H)^{-1} H^T$. Using (5) the instrumental identification criterion (IIC) may be written in the form

$$\mathcal{J}_\varepsilon(\theta, K) = \mathbb{E}\{\varepsilon_k^T(\theta) \varepsilon_k(\theta)\}. \tag{6}$$

The following theorem holds (see [1] for more details).

Theorem. Let matrices H and B in (2) be independent on θ and conditions (3) are satisfied. Then $\mathcal{J}_e(\theta, K)$ and $\mathcal{J}_\varepsilon(\theta, K)$ have one and the same minimizer and the following relation holds

$$\mathcal{J}_\varepsilon(\theta, K) = \mathcal{J}_e(\theta, K) + \text{Const}. \tag{7}$$

Considering the corresponding equations of both filters, the second term on the right-hand side of (5) can be rewritten as follows:

$$\hat{x}_k^*(\theta) = Y_{k|k-1}^{-1} y_{k|k-1} + B T_{k-1}^{-1} t_{k-1}. \tag{8}$$

Finally, the expression for $\mathcal{J}_\varepsilon(\theta, K)$ can be written in terms of an information filter as follows:

$$\mathcal{J}_\varepsilon(\theta, K) = \frac{1}{K} \sum_{k=1}^{K} \varepsilon_k^T(\theta) \varepsilon_k(\theta) \tag{9}$$

where

$$\varepsilon_k(\theta) = W^+ z_k - (Y_{k|k-1}^{-1} y_{k|k-1} + B T_{k-1}^{-1} t_{k-1}). \tag{10}$$

Here $Y_{k|k-1}$ is an information matrix and $y_{k|k-1}$ is an information vector obtained on the time update step, T_{k-1} and t_{k-1} are obtained on the input estimation step of the information algorithm.

The proposed approach can be also applied for systems with both known and unknown inputs, i.e., when the original system has the form

$$\begin{cases} x_k = F x_{k-1} + B d_{k-1} + C u_{k-1} + G w_k, \\ z_k = H x_k + v_k, \quad k = 1, 2, \ldots, K \end{cases} \tag{11}$$

where u_k is known input vector (in this case time update steps of both algorithms have to be modified).

The application of the identification criterion $\mathcal{J}_\varepsilon(\theta, K)$ based on the GDM algorithm was demonstrated in [1] for a model of object motion on a plane. In the next section we consider another parameter identification problem and compare the identification results for covariance and information versions of the identification criterion.

4. Diffusion model

Parabolic equations in combination with Kalman-type filtering methods are widely used for modeling control processes of optical systems [8], heat and mass transfer processes [9], in environmental monitoring [10] and in other fields.

Let us consider the problem of identifying the diffusion coefficient α of a one-dimensional diffusion model

$$\frac{\partial c(x,t)}{\partial t} = \alpha \frac{\partial^2 c(x,t)}{\partial x^2} \tag{12}$$

with known initial condition

$$c(x,0) = \varphi(x) \tag{13}$$

and unknown boundary conditions of the first kind

$$c(a,t) = f(t), c(b,t) = g(t) \tag{14}$$

where $(x,t) \in [a,b] \times [0,T]$.

For this problem the unknown parameter $\theta = \alpha$ is a scalar. To obtain the discrete linear system (2) from the original continuous model (12 – 14) a discretization process is usually used (see, for example, [9 – 12]).

Consider in the plane Oxt a uniform grid

$$\{(x_i, t_k) | i = 0,1,\dots,N, k = 0,1,\dots,K\}, \tag{15}$$

where $x_i = a + i\Delta x, t_k = k\Delta t, \Delta x = \frac{b-a}{N}, \Delta t = \frac{T}{K}$.

Denote $c_i^k = c(x_i, t_k)$, $\varphi_i = \varphi(x_i)$, $f^k = f(t_k)$, $g^k = g(t_k)$.

Let us replace in (12) the partial derivatives with their finite-difference approximations using FTCS (forward-time and centered-space) scheme:

$$\frac{c_i^k - c_i^{k-1}}{\Delta t} = \alpha \frac{c_{i+1}^{k-1} - 2c_i^{k-1} + c_{i-1}^{k-1}}{\Delta x^2}, \quad i = 1,2,\dots,N-1, k = 1,2,\dots,K. \tag{16}$$

Note that alternative approximation schemes may be used, especially when (12) contains additional terms like convection, etc.

Discretization of the initial condition (13) yields:

$$c_i^0 = \varphi_i, i = 0,1,\dots,N. \tag{17}$$

For boundary conditions (14) we obtain:

$$c_0^k = f^k, c_N^k = g^k, k = 1,2,\dots,K. \tag{18}$$

Denote $r = \frac{\alpha\Delta t}{\Delta x^2}$, $a_1 = r$, $a_2 = 1 - 2r$, then the first equation of system (2) may be written as follows:

$$\begin{bmatrix} c_1^k \\ c_2^k \\ \cdots \\ c_{N-2}^k \\ c_{N-1}^k \end{bmatrix} = \begin{bmatrix} a_2 & a_1 & \cdots & 0 & 0 \\ a_1 & a_2 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_2 & a_1 \\ 0 & 0 & \cdots & a_1 & a_2 \end{bmatrix} \begin{bmatrix} c_1^{k-1} \\ c_2^{k-1} \\ \cdots \\ c_{N-2}^{k-1} \\ c_{N-1}^{k-1} \end{bmatrix} + \begin{bmatrix} a_1 f^{k-1} \\ 0 \\ \cdots \\ 0 \\ a_1 g^{k-1} \end{bmatrix} \tag{19}$$

where matrix $F = \mathrm{tridiag}(a_1, a_2, a_1)$ depends on the unknown parameter $\theta = \alpha$, and the state vector consists of elements corresponding to internal nodes of the spatial grid (see [13 – 15] for more details).

To complete the process of obtaining the discrete linear system (2), we substitute into (19) the state noise w_k with a given covariance matrix Q to account for model uncertainties and add the measurement equation:

$$\begin{cases} x_k = F(\theta)x_{k-1} + d_{k-1} + w_k, \\ z_k = x_k + v_k, \quad k = 1,2,\dots,K \end{cases} \tag{20}$$

(we take $H = I$ to meet the condition rank $H = n$).

Remark 1. In the case of a non-homogeneous equation (12)

$$\frac{\partial c(x,t)}{\partial t} = \alpha \frac{\partial^2 c(x,t)}{\partial x^2} + F(x,t), \tag{21}$$

where $F(x,t)$ is known, the same discretization scheme can be used, but the result is a discrete stochastic system of the form (11).

Remark 2. In the case of a variable diffusion coefficient $\alpha = \alpha(x)$, a half-step discretization scheme can be used. In this case, the unknown parameter θ will be vector.

The identification criterion (9) can thus be applied to estimate the diffusion coefficient. Note that covariance-based versions of the GDM algorithm were employed for similar problems in [13 – 15]. The next section provides a comparison between the covariance and information versions of the identification criterion.

5. Numerical example

Let $(x,t) \in [0,1] \times [0,2]$, the true value $\alpha = 0.3$, the initial condition

$$c(x,0) = 10x(1-x) \tag{22}$$

and boundary conditions

$$c(0,t) = \frac{t^2}{2}, c(1,t) = 0. \tag{23}$$

Let us take a grid with 13 nodes along the Ox axis (the state vector consists of 11 elements) and 401 nodes along the Ot axis, i.e. $\Delta x \approx 0.083, \Delta t = 0.005$. Note that in this case, the Neumann stability condition $\frac{\alpha \Delta t}{(\Delta x)^2} \leq \frac{1}{2}$ for the finite-difference scheme is satisfied. Let $Q = 10^{-3} I_{11}$. Consider the measurement scheme with $H = I_{11}$, $R = \delta I_{11}$.

Figure 1 presents the numerical solution to the direct problem, Figure 2 depicts the noisy measurements for $\delta = 0.01$, and Figure 3 displays the curves of the covariance (GDM) and information (IGDM) identification criteria for $\delta = 0.01$, averaged over 50 independent runs. As can be seen from Figure 3, the curves for both criteria coincide.

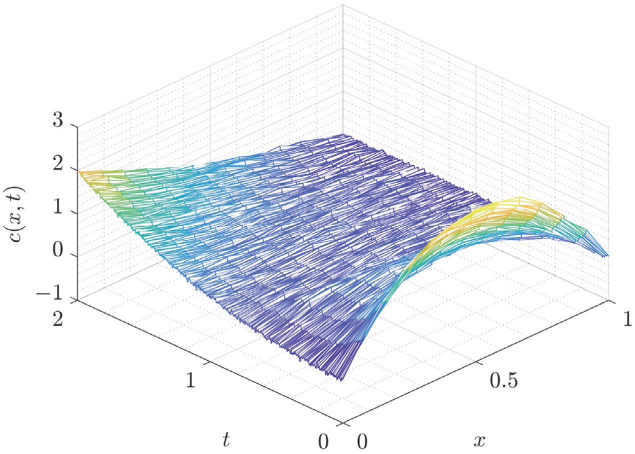

Fig. 1. Problem solution

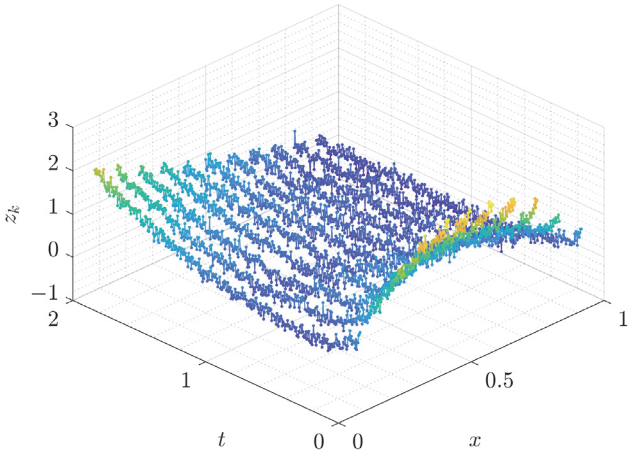

Fig. 2. Noisy measurements

The identification process was modeled in the MATLAB R2017a environment. To minimize the identification criterion, the unconstrained minimization function *fminunc* from the Optimization Toolbox was used. Objective functions to calculate covariance and information versions of the identification criterion were implemented. The initial values were randomly selected from the interval [0, 1].

Table 1 presents the results of the diffusion coefficient identification for both versions of the identification criterion across different values of δ, averaged over 50 experiments. The results indicate that the information-based criterion achieves comparable accuracy to the covariance version while being approximately 1.4 times slower.

Conclusion

This study addresses the problem of parameter identification for discrete linear stochastic systems with unknown inputs. The key results of the work include:

1. Development of an information-based identification criterion.

A novel criterion was introduced using the information form of the Gillijns – De Moor algorithm. This approach is particularly valuable when the covariance version is impractical or undesirable, such as in cases where prior knowledge of covariance matrices is unavailable. Another important advantage of information-type algorithms is that they serve as a natural basis for building distributed estimation and identification methods in multisensor systems [16].

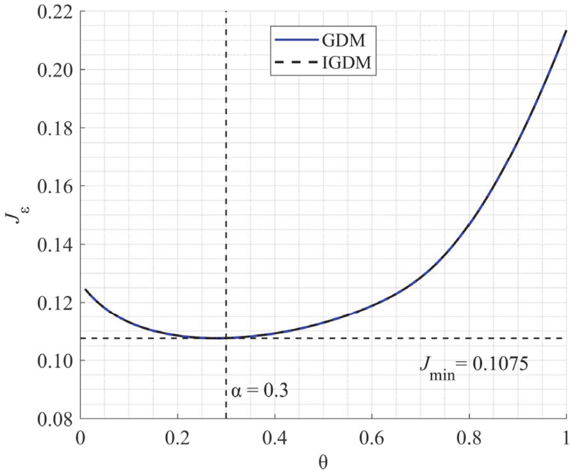

Fig. 3. Averaged identification criteria

Tab. 1. Identification results

δ	GDM			IGDM		
	Mean	**RMSE**	**Time (s)**	**Mean**	**RMSE**	**Time (s)**
0.010	0.280409023137	0.021277543520	0.3123	0.280409020201	0.021277549890	0.4557
0.005	0.285653205826	0.016084145629	0.3344	0.285653205012	0.016084147557	0.4878
0.001	0.296994093613	0.006155273013	0.3640	0.296994099981	0.006155268437	0.5186

2. Numerical validation.

Computational experiments for a one-dimensional diffusion model demonstrate the possibility of applying the proposed approach to identifying the diffusion coefficient in the presence of state and measurement noise. The results confirmed that the information-based criterion achieves the same precision as the covariance version, supporting the theoretical findings.

The proposed identification criterion extends the results recently obtained in [1]. It allows estimating unknown parameters of a system even without information about external input data, which is crucial for practical application. Further research will be aimed at constructing numerically stable and distributed modifications of the proposed criterion.

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