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DESIGN OF TWO-MIRROR SYSTEMS

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Abstract—Methods are surveyed for the design of mirror systems which transform, within the geometrical optics approximation, a primary spherical wave into a wave with specified ray structure (namely flat, spherical, or with a specific focal line). The existing approaches to this problem are discussed in detail. Mirror surfaces can be chosen starting from different conditions: (a) the amplitude distribution over the initial wave front is transformed into a specified amplitude distribution over the front of the doubly reflected wave; (b) focusing (without aberrations) of two waves from different directions; (c) the polarization structure of the primary wave is transformed into the specified polarization structure of a doubly reflected wave. For different formulations, the problem boils down to solving equations in second- or first-order partial derivatives, and equations in partial derivatives with delayed argument. In degenerate situations (the flat or the axially symmetric problem), the solution reduces to ordinary differential equations. Boundary conditions providing unique solutions are considered.

Unlike classically constructed optical devices in the visible range, optical-type devices (antennae) working in the centimetre and millimetre bands may utilize nonspherical surfaces. This circumstance is a result of available production technologies, because manufacturing tolerances are determined in part by the wavelength. The present-day limit of production accuracy of non-spherical surfaces with linear dimensions of 10–30 m is 0.05 mm, such surfaces being produced by computer controlled machines. The manufacture of such unusual surfaces in optics opens up novel possibilities. Our report is devoted to a description of various types of such devices and to a short exposition of their design principles.

The most important requirement imposed on optical devices is the attainment of maximal directivity (resolving power). This in turn demands zero aberration, i.e. the satisfaction of a phase condition. Reducing aberration in a particular direction to zero (equivalent to point focusing at a finite distance), given that the primary field is a spherical wave, may be achieved by means of a single-mirror system whose surface is an ellipsoid of revolution. In the special, but practically important case of focusing at infinity (required of communication, location or other antennae), the mirror surface must be a paraboloid of revolution. Single mirror antennae are thus adequate in generating the required phase structure, but clearly cannot cope with regulating the amplitude structure of secondary wavefronts, or with controlling polarizations. We recall that just to eliminate surface aberrations the mirror has to be aspheric (though it is in fact a surface of revolution).

Two-mirror antennae have great possibilities. By suitable choice of the two mirror surfaces different goals may be realized: one can either regulate the amplitude distribution of the focused field, or eliminate aberration at two focal points (bifocal system).

Let us first discuss how amplitude regulation can be achieved. The simplest solution is obtained for an axisymmetric system (see Fig. 1). The problem is solved in two stages: first, via energy considerations, one sets up the correspondence between beams of the primary spherical wave and the secondary plane wave, i.e. the function $r = r(\theta)$. This correspondence is of course antisymmetric for the meridional cross-section. Then in the second stage one determines, via the mirror reflection conditions, the transformed beams formed on the mirrors [1–3]. This type of two-mirror system significantly reduces energy losses across the mirror edges, and at the same time equalizes the amplitude distribution (increasing the resolving power) along the plane wavefront. Such systems are extensively utilized in present-day antenna technology and admit a uniformity value of distribution of up to 0.9 [4]. The mirror profile is determined by a first-order ordinary differential equation which can be solved by quadratures [1, 2].

The theory of the synthesis of non-axisymmetric mirrors is considerably more complicated. These systems allow one to eliminate vignetting of large mirrors by small ones [3, 5, 6]. The transformation of a spherical into a plane wave at specified amplitude distributions of the primary and secondary waves is given by the solution of a nonlinear partial differential equation (a Monge–Ampere type equation) [5]. The theory of such equations has not yet been worked out and the boundary

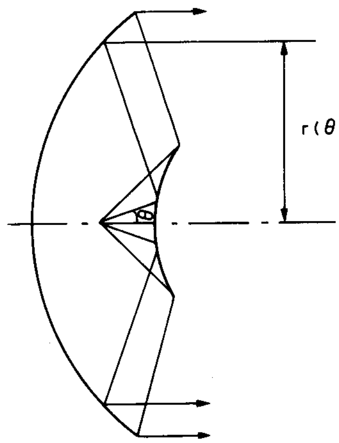

Fig. 1.

conditions required to ensure existence and uniqueness of solutions are still the subject of discussion [7, 8].

It is usually assumed that for this transformation one need only demand that a finite segment T_1 of the primary front be transformed into a finite segment T_2 of the doubly reflected wave, such that the contour Γ_1 is transformed into contour Γ_2, without imposing the condition expressing the correspondence between points of Γ_1 and Γ_2. This requirement was used in [5] as the equivalent of a boundary condition. However, the inadequacy of such a procedure may be shown by a simple example. Let T_1 and T_2 be circular sections of a plane wave with uniform amplitude distribution. Two-mirror antennae transforming T_1 and T_2 into each other may be constructed by three different methods:

(1) two plane mirrors,
(2) two confocal paraboloids of rotation,
(3) two confocal parabolic cylinders.

All three alternatives satisfy the conditions of the problem, yet each will have different rules for intercontour point matching. For example, in the third variant the angle of orientation of the parabolic cylinder is clearly arbitrary. This example demonstrates that the boundary condition must be given in association with the correspondence rule between points of Γ_1 and Γ_2.

If energy conditions are required to be satisfied only in the mean, rather than strictly (along a single azimuth coordinate), then the problem reduces to a nonlinear equation in the first-order partial derivatives [7, 8], whose theory is well established. It leads one into solving a set of five ordinary differential equations (by the method of characteristics). By modifying somewhat the method described (while preserving the single-parameter nature of the amplitude transformation), one of the mirrors may be made into the technically more accessible axisymmetric form. The design of such a mirror is through the solution of a second-order ordinary differential equation.

Two-mirror antennae can be used to transform not only amplitudes but also polarizations. However, if the mirror surfaces are smooth, then this type of system will only perform a rotation of the polarization ellipse, without being able to vary its eccentricity [9]. In order to change the polarization ellipse's eccentricity anisotropic (lattice) mirrors must be used. In this case, by regulating the anisotropy tensor parameters, we can control the shifts of the polarization ellipse, while by adjusting the anisotropic mirror surfaces we can alter the amplitude distribution at the output of the system. The presence of anisotropy on both surfaces allows one to moderate the anisotropy requirement of individual mirrors [10].

Two-mirror systems may also be utilized in aberration-free focusing into two specified directions. Oddly enough, the mathematical problem of determining the mirror surfaces in this case is simpler than for the amplitude transformation. The mirror surfaces are determined by a set of recursive relations, in which the initial segment of one of the mirrors may be chosen arbitrarily [11–13]. The method for solving the two-dimensional problem is given in [11], while the three-dimensional problem is tackled in [13] and [14]. If the system is designed to focus at points in two directions, then in any intermediate direction the aberration will be quite small [15]. Such systems have the advantage of a wider field of view compared with visible-range optical systems. For a fair comparison one must naturally assume that in both cases the transformations are carried out by two surfaces.

As the number of reflections or refractions increases so does the number of possible aberration-free directions. This sort of approach is equivalent to using polynomial interpolations having a set of zeroes in the appropriate solid angle. The determination of surfaces for an N-focal system reduces to the determination of N reflecting or refracting surfaces. The solution is given in terms of algebraic equations [16, 17].

From the standpoint of HF techniques all the foregoing are antenna systems operating in the reception and/or transmission mode. Recently wireless energy transmission systems, so called beam systems, have become widespread. These too are designed on geometrical optics principles, which means that their separate elements are positioned in neighbouring or Fresnel regions with respect to one another. Their design is based either on the laws of geometrical optics or generalizations thereof, partly using diffraction ideas together with Gaussian beam theory [18].

Specific optical-type systems may also be used to design focusing systems, in particular, systems focusing along a line rather than into a point (coherent focusers). In the focusing of the primary spherical or plane wave into a given focal line, one of the mirrors suffices to regulate the amplitude along the focal line [19–23]. The solution to the foregoing problem depends on what is taken as the intensity field along the focal line. If for the intensity one uses, instead, the energy flux corresponding to the maximum intensity of the "principal lobe", then the solution is as in [19], whereas if the intensity is taken as the square of the amplitude along the focal line, then the solution will be as in [22].

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