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DIFFRACTION-ELEMENT OPTICAL SYSTEMS WITH MULTIPLE FIELD OF VIEW

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Abstract—The paper deals with optical systems having multiplexed field of view that are designed for optical data processing, robotic vision, etc. The constituent components include two phase-relief diffraction elements, a grating and a lens. Problems involved in formulating the system are discussed. Relations are derived for paraxial raytracing and aberration control of the generic diffraction elements.

Some systems of data processing, robotic vision, etc. call for optical systems with multiple fields of view. Such systems must ensure spatial matching of the images of equidistant objects viewed at different angles. Physically, such optical systems consist of two basic components, a "demultiplexer" that executes beam matching, and a lens that forms the image.

The simplest demultiplexer is a diffraction grating that forms beams of equal intensity in different diffraction orders [1]. If such a system uses a single diffraction lens as an objective, then this combination may be fabricated as a single optical device, a plane-parallel plate with linear and annular phase relief diffraction structures engraved on the opposite faces. The performance of such a plate in any order of the diffraction grating except zero is equivalent to that of an off-axis holographic optical element (HOE). Focusing and aberration properties of an off-axis HOE have been discussed by Champagne [2]. Recognizing that radiation detectors are designed as a rule for coupling with optical systems that form a plane image, the present investigation is devoted to the focusing and aberration properties which an off-axis HOE exhibits in forming an image on a plane normal to the axis of vision.

Figure 1 presents a general holographic system for formation of an off-axis HOE for distinct object imagery. At the recording wavelength λ_0 the HOE will form an aberration-free image of an infinitely distant point source in the direction 00_1. This line is the axis of the collimated recording beam and will be referred to as the axis of the HOE directional pattern at the wavelength λ_0. The axis $00'$ will be referred to as the axis of vision, the point $0'$ where the aberration free image is formed will be referred to as the HOE focus, and the distance $00' = f_0$ as the focal length of the element.

If a distant point source is not on the axis of the directional pattern, or its emission λ is other than λ_0, its image shifts from point $0'$ and is distorted by aberrations. The HOE raytracing equations that determine the paraxial coordinates of the image and estimate aberrations may be obtained by the method outlined by Bobrov *et al.* [3].

In the XYZ system of coordinates centred at the HOE aperture centre, such that the YOZ plane passes through the axes of directional pattern and vision, and the axis is normal to the HOE plane (Fig. 2), the equations relating the direction cosines of incident (m_x, m_y, m_z) and diffracted (m'_x, m'_y, m'_z) rays have the forms

$$\left.\begin{aligned} m'_x &= m_x - \mu\Omega x, \\ m'_y &= m_y - \mu[\sin\beta_1 + \Omega(y - f'_0 \sin\beta_2), \\ m'_z &= \sqrt{1 - m'^2_x - m'^2_y}), \end{aligned}\right\} \tag{1}$$

where

$$\Omega = \frac{1}{f'_0}\left(1 + \frac{x^2 + y^2}{f'^2_0} - \frac{2y\sin\beta_2}{f'_0}\right)^{-1/2}, \tag{2}$$

and $\mu = \lambda/\lambda_0$.

Equations (1) were derived on the assumption that the HOE operates in the -1th order of diffraction and is in air.

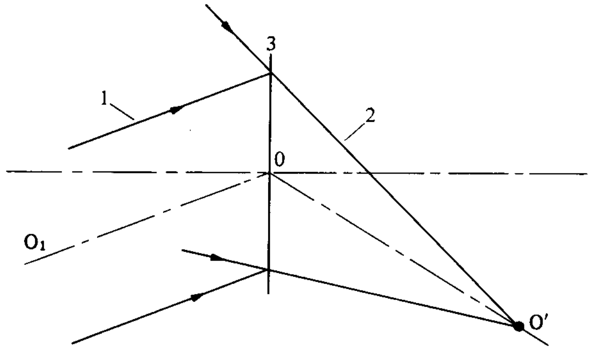

Fig. 1. Holographic systems for forming an extra-axial HOE: (1), (2) collimated and converging recording beams, (3) plane of recording.

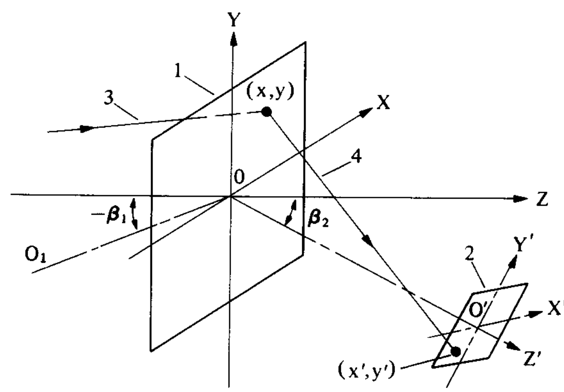

Fig. 2. Nomenclature for raytracing an off-axis HOE: (1) plane of HOE diffraction structure, (2) focal plane, (3) incident and diffracted rays.

In the $X'Y'Z'$ system of coordinates attached to the focal plane (normal to the axis of vision and at a distance f'_0 from the centre of HOE aperture) and the axis of observation (Fig. 2), the coordinates of a point where the diffracted ray pierces the focal plane have the form

$$\left.\begin{aligned} x &= x + bm'_x \\ y' &= y + b(m'_y \cos\beta_2 - m'_z \sin\beta_2), \end{aligned}\right\} \tag{3}$$

where

$$G = \frac{f'_0 - y \sin\beta_2}{m'_y \sin\beta_2 + m'_z \cos\beta_2}. \tag{4}$$

Denote by $\xi(\xi')$ the tangent of the angle between the incident (diffracted) ray and the YOZ plane, i.e. the meridional plane, and by $\eta(\eta')$ the tangent of the angle between the incident (diffracted) beam and the axis of the HOE directional pattern (axis of observation) measured in the meridional plane. The paraxial equations and equations for aberration coefficients may be obtained by expanding Eqs (1)–(4) in series of powers of x, y, ξ, η truncated at the terms of a suitable order of smallness. In the paraxial approximation, the series stop at the first-order terms, so that from (1)–(4) it follows:

$$\left.\begin{aligned} x' &= f'_0\xi + M^{cr}_{1000}x, \\ y' &= f'_0\eta + M_{0001}\eta + M^{cr}_{0000} + M^{cr}_{0100}y, \end{aligned}\right\} \tag{5}$$

where

$$\left.\begin{aligned} M_{1000}^{cr} &= -\frac{\Delta\lambda}{\lambda_0}, \\ M_{0000}^{cr} &= -f'_0\left(\frac{\Delta\lambda}{\lambda_0}\right)\left(\frac{\sin\beta_1 - \sin\beta_2}{\cos\beta_2}\right), \\ M_{0100}^{cr} &= -\left(\frac{\Delta\lambda}{\lambda_0}\right)\cos\beta_2, \\ M_{0001} &= -f'_0\left(1 - \frac{\cos\beta_1}{\cos\beta_2}\right). \end{aligned}\right\} \quad (6)$$

The first three coefficients in (6) determine the chromatic aberrations of the off-axis HOE: M_{1000}^{cr} is the sagittal first-order chromatic aberration, M_{0000}^{cr} and M_{0100}^{cr} are the meridional chromatic aberrations of zero and first order, respectively. The M_{0001} coefficient characterizes the anamorphic properties of the off-axis HOE.

In determining the primary monochromatic aberrations, it is important to note that, for optical systems without an axis of symmetry, the primary aberrations are those of second-order. Assuming $\mu = 1$ in Eqs (1), expanding (1)–(4) in power series, and retaining the second-order terms in x, y, ξ and η we obtain the expression for primary monochromatic aberrations:

$$\begin{aligned} \Delta x'_2 &= M_{0110}\, y\xi; \\ \Delta y'_2 &= M_{1010}\, x\xi + M_{0101}\, y\eta + M_{0020}\,\xi^2 + M_{0002}\,\eta^2 \end{aligned} \quad (7)$$

where

$$\begin{aligned} M_{0110} &= -\sin\beta_2, \\ M_{1010} &= \tan\beta_2, \\ M_{0101} &= -2\cos\beta_1 \tan\beta_2, \\ M_{0020} &= \frac{1}{2} f'_0 \tan\beta_2, \\ M_{0002} &= \frac{1}{2} f'_0\left(\frac{\cos^2\beta_1 \sin\beta_2 - \sin\beta_1 \cos^2\beta_2}{\cos\beta_2}\right). \end{aligned} \quad (8)$$

Following the nomenclature of Rusinov [4] we name these aberration coefficients

M_{0110}, M_{1010}, M_{0101} as astigmatic coefficients, and

M_{0002}, M_{0020} as coefficients respectively of scale and parabolic distortion.

The above expressions may be used as a basis for analysis and design of optical systems involving off-axis HOEs, and, in particular, of diffraction systems with multiple-sector fields of view.

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