On the synthesis of the efficient algorithm under the set of convolution algorithms

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Abstract:

The paper deals with the problem of development of an efficient algorithm designed for the purpose of linear convolution calculation. To develop the desired algorithm, a closure of a predetermined set of algorithms is introduced according to the model (transformation), and it represents a new set of algorithms. The algorithm with the best computational characteristics of all the closure is called an induced algorithm. The induced algorithm, by construction, uses not only the most appropriate subset of algorithms of the original set, but also the characteristics of the processed signal with impulse response for the purpose of calculating the convolution. In this paper a number of theorems is proved that establish necessary and sufficient conditions for the efficiency and strict efficiency of the induced algorithm. Similar theorems are proved for a practically important case when the algorithms of the main classes are chosen as the initial set: the algorithm of direct calculation of convolution; algorithms built on the basis of discrete orthogonal transformations (FFT type); and recursive convolution calculation algorithms (recursive filters). A general description of the method for the development an efficient algorithm on the basis of the theoretical results obtained, is provided. A detailed algorithmic description of the procedures that implement the individual steps of the proposed method is presented. Several well-known convolution calculation algorithms are presented, which are particular solutions to the problem of the development of an efficient algorithm.

<u>Keywords</u>: Convolution Algorithm, linear convolution, induced algorithm, discrete orthogonal transformation, recursive filters

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Access full text (in Russian)

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